While you are waiting...

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Simulating Language
Lecture 10: Iterated Bayesian Learning

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Summary and next up

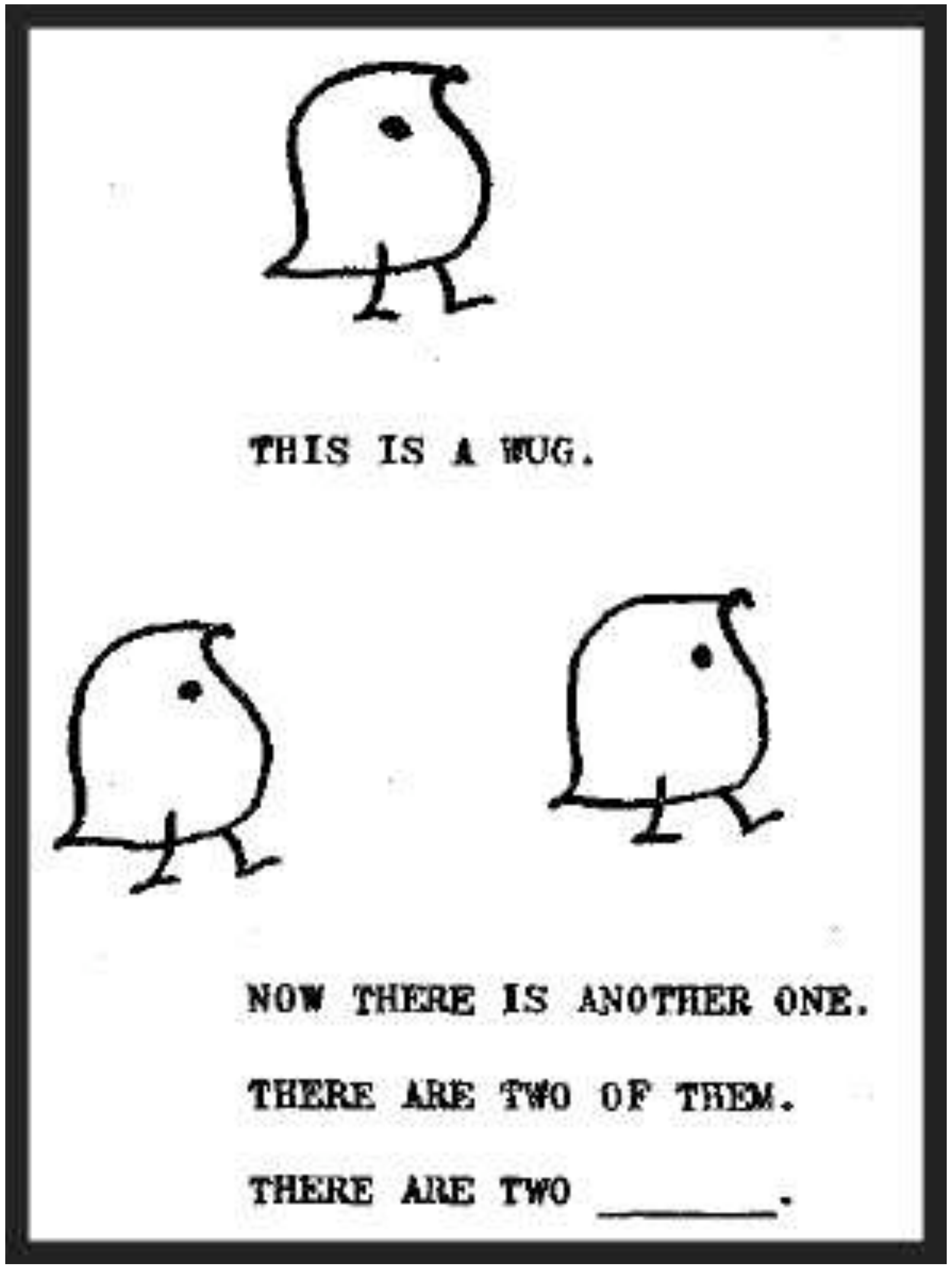
$$P(h|d) \propto P(d|h)P(h)$$

- Bayesian learning: a nice simple way to model learning
- Involves probabilities:
    - For each possible language, what is its prior probability? What is the likelihood of the linguistic data if people are using that language?
- Make the bias of learners beautifully explicit

Variation in language

- **An observation:** languages tend to avoid having two or more forms which occur in identical contexts and perform precisely the same functions
- Within individual languages: phonological or sociolinguistic conditioning of alternation
- Over time: historical tendency towards analogical levelling

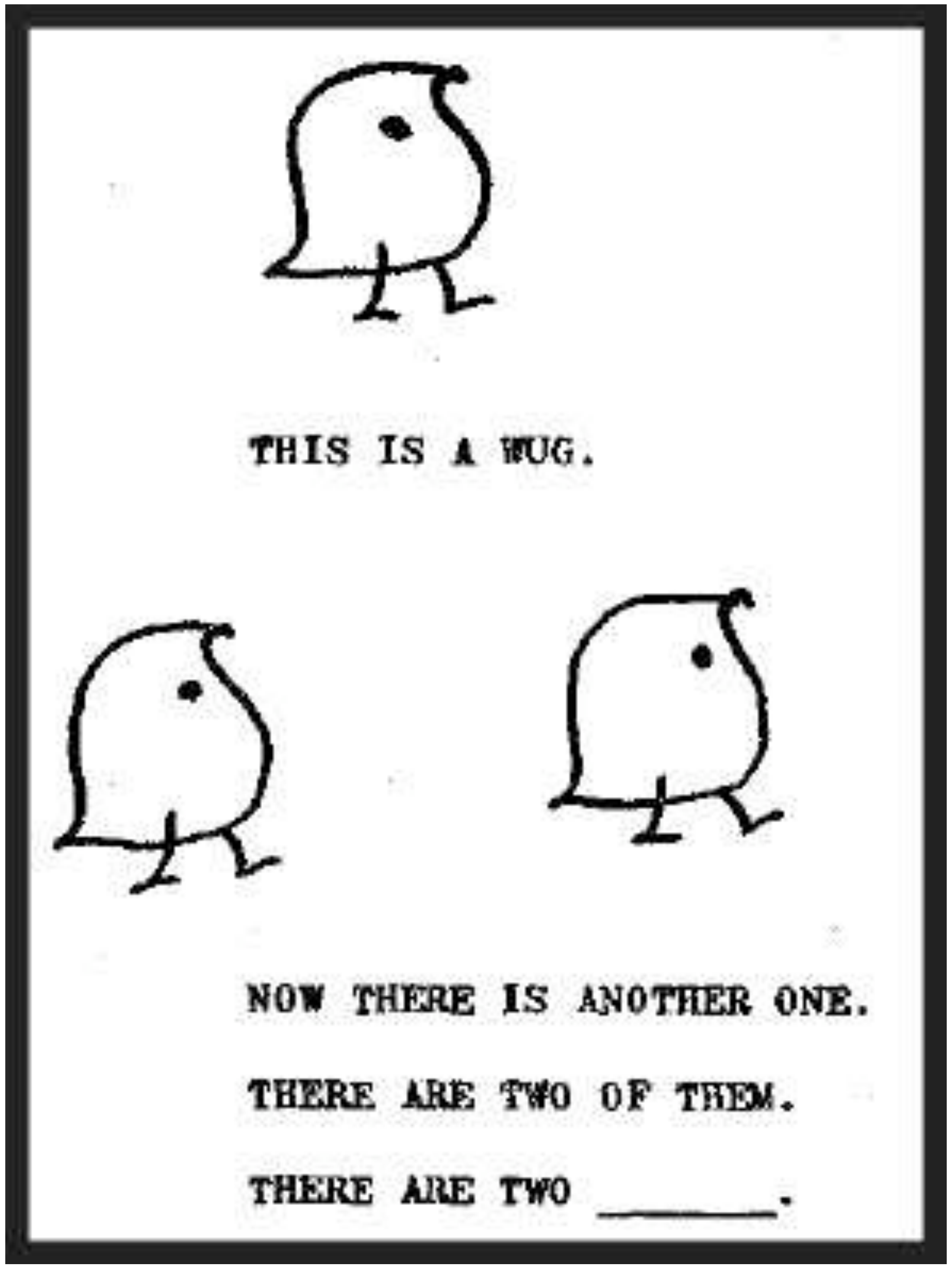
The wug test



- **“wugs”**
- Not “wugen”
  - ox, oxen
- Not “wug”
  - sheep, sheep
- Not “weeg”
  - foot, feet

These ways of marking the plural are relics of older systems which have died out: **loss of variability**

The wug test continued



- Three allomorphs for the regular plural, conditioned on phonology of stem
  - One wug, two /wʌgz/
  - One wup, two /wʌps/
  - One wass, two /wasəz/
- **Conditioning** of variation

Variation in language

- **An observation:** languages tend to avoid having two or more forms which occur in identical contexts and perform precisely the same functions
- Within individual languages: phonological or sociolinguistic conditioning of alternation
- Over time: historical tendency towards analogical levelling
- **During development:** Mutual exclusivity; overregularization of morphological paradigms

A prediction about the bias of learners

- Languages tend not to exhibit free (unpredictable, unconditioned) variation
- Languages are transmitted via iterated learning, and should reflect the biases of learners
- We already know that child learners are biased against ‘variation’ in the lexicon (synonymy, Mutual Exclusivity)
- This kind of learning bias is probably pretty widespread, right?

An artificial language learning study

Hudson-Kam & Newport (2005)

- Adults trained and tested on an artificial language
    - 36 nouns, 12 verbs, negation, **2 determiners**
- Multiple training sessions
- Variable (unpredictable) use of ‘determiners’

An artificial language learning study

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flern	blergen	**(ka)**	flugat	**(ka)**
rams	elephant	**(Det)**	giraffe	**(Det)**

“the elephant rams the giraffe”

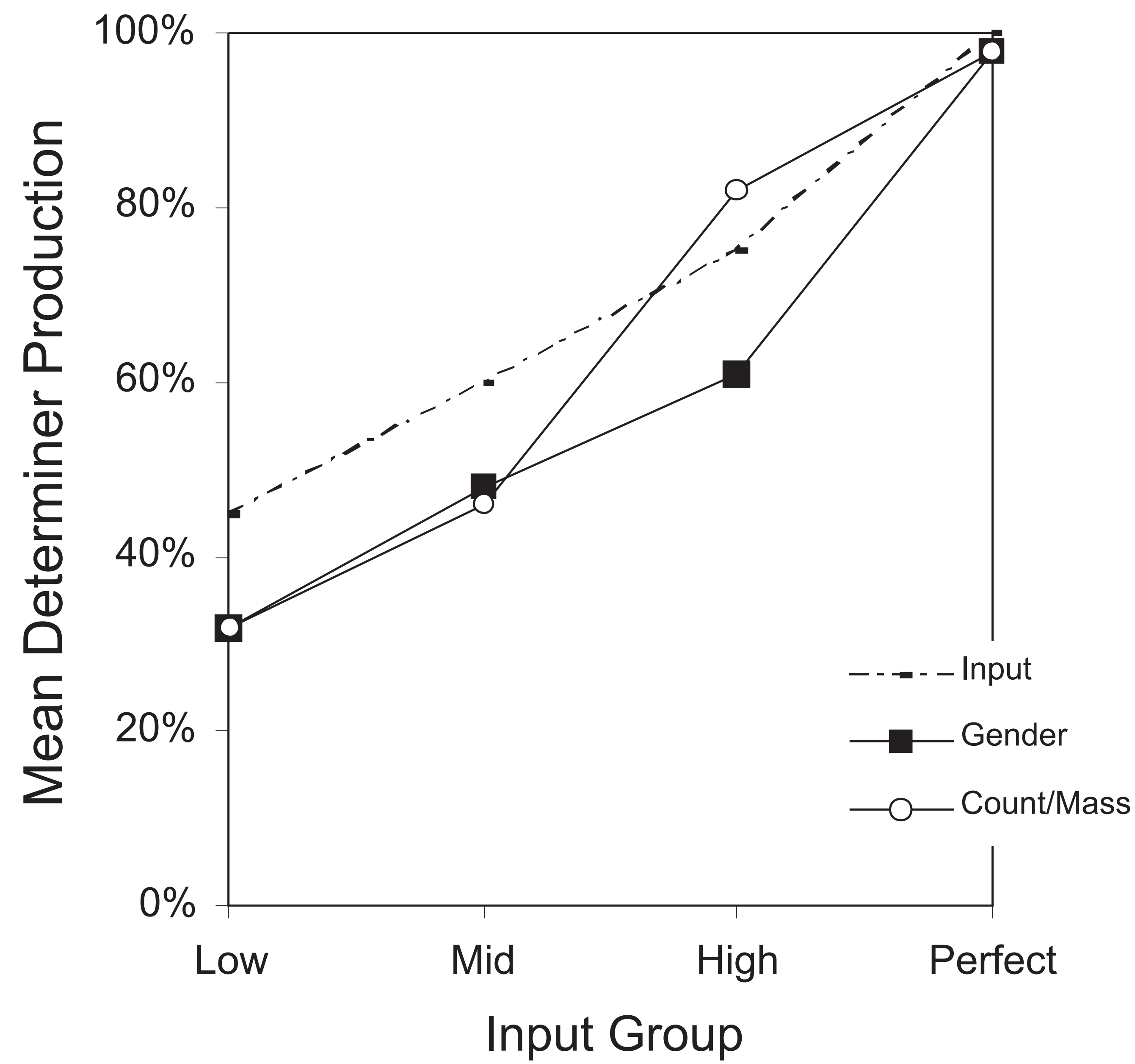
Adults **probability match**

- If trained on variable input, produce variable output

- Does this mean they have the 'wrong' bias to explain how language is?

- Or do we just have bad intuitions about how a biased learner should behave?

- We need a model
  - Reali & Griffiths (2009)



The model in a nutshell

- Let's simplify: one grammatical function, two words which could mark it
    - word 0, word 1
- The learner gets some data
    - word 0, word 0, word 1, word 1, word 0, ...
    - ∅, ∅, ka, ka, ∅, ...
- And has to infer how often it should use each word
    - "I will use word 0 60% of the time, and word 1 40% of the time"
    - "I will use word 1 40% of the time"
    - $\theta = 0.4$

A little more detail

$$P(h|d) \propto P(d|h)P(h)$$

- The learner gets some data, d
  - word 0, word 0, word 1, word 1, word 0, ...
- And has to infer how often it should use each word, based on that data
  - θ
- The learner will consider several possible hypotheses about θ
  - Is word 1 being used 5% of the time? 15%? 25%? ...
  - θ = 0.05? θ = 0.15? θ = 0.25? ...
- The learner will use Bayesian inference to decide what θ is

$$P(\theta|d) \propto P(d|\theta)P(\theta)$$

The likelihood

- Let's say that the probability of using word 1 is 0.5 - both words are equally likely to be used
  - $\theta = 0.5 = 1/2$
- Let's say your data consists of a single item: a single occurrence of word 1
  - $d = [1]$
- What is the likelihood of this data, given that $\theta = 0.5$?
  - What is $p(d = [1] \mid \theta = 1/2)$?

The likelihood

- What is $p(d = [1,1,1] \mid \theta = 1/2)$?

A. 0

B. 1

C. 1/2

D. 1/8

E. 7/8

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The likelihood

- What is $p(d = [1,1,1] \mid \theta = 3/4)$?

A. 0

B. 1

C. 3/4

D. 1/64

E. 27/64

The likelihood

- What is $p(d = [1,1,1] \mid \theta = 1/10)$?

A. 0

B. 1

C. 1/10

D. 1/100

E. 1/1000

The likelihood: summary

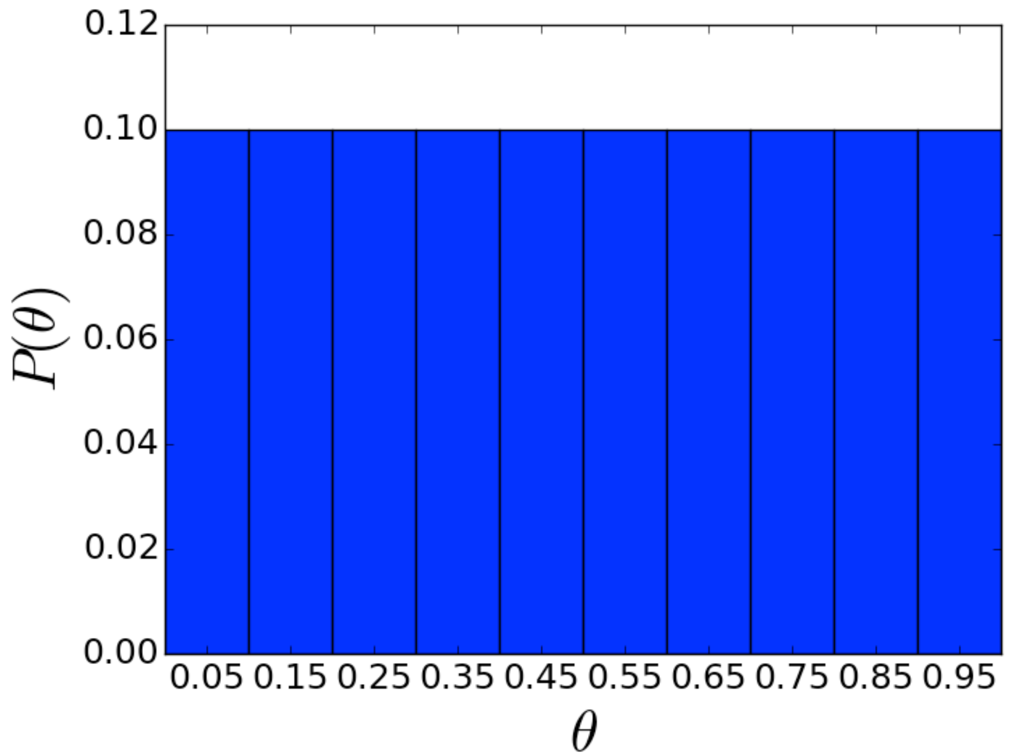
- When θ is high, data containing lots of word 1 is very likely
- When θ is around 0.5, data containing lots of word 1 is not that likely
  - A mix of 1s and 0s is more likely
- When θ is low, data containing lots of word 1 is very unlikely
  - Lots of word 0 is more likely

The prior

- Let's say our learner considers 10 possible values of θ
    - 0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95
- Our prior is a probability distribution: for each possible value of θ, we have to say how likely our learner thinks it is, before they have seen any data
    - High prior probability for a given value of θ means, before seeing any data, the learner thinks that value is likely
    - Low prior probability for a given value of θ means, a priori, the learner thinks that value is unlikely

Which of these possible priors would be a good model for an **unbiased learner**, who thinks each possible value of θ is equally probable a priori?

A



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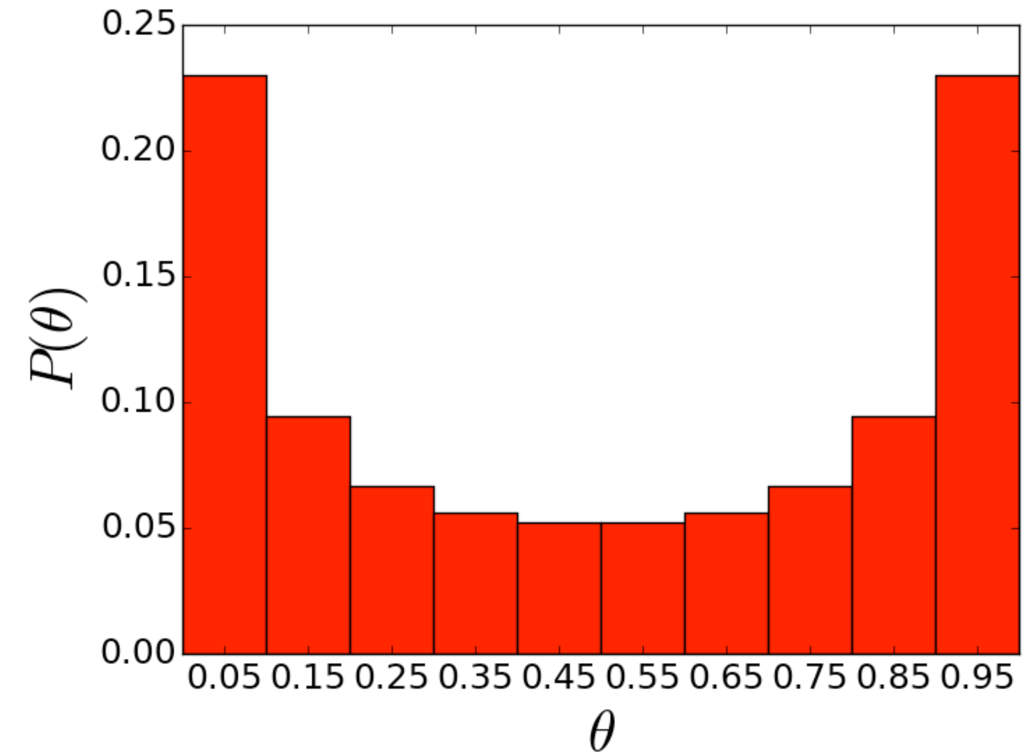

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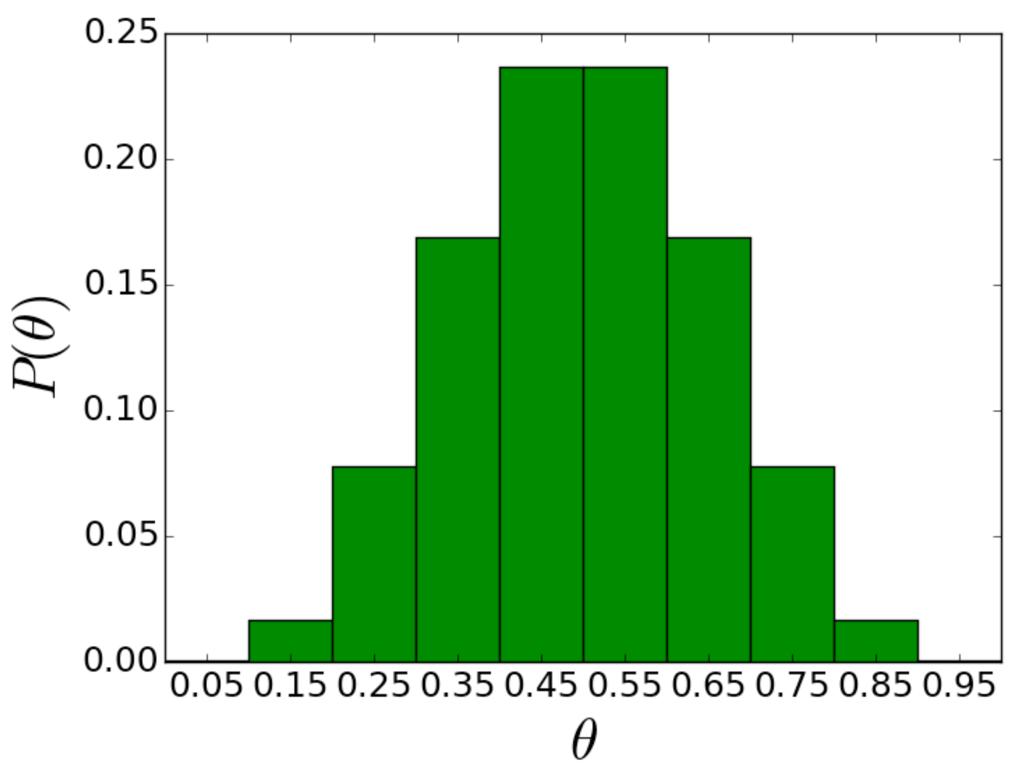
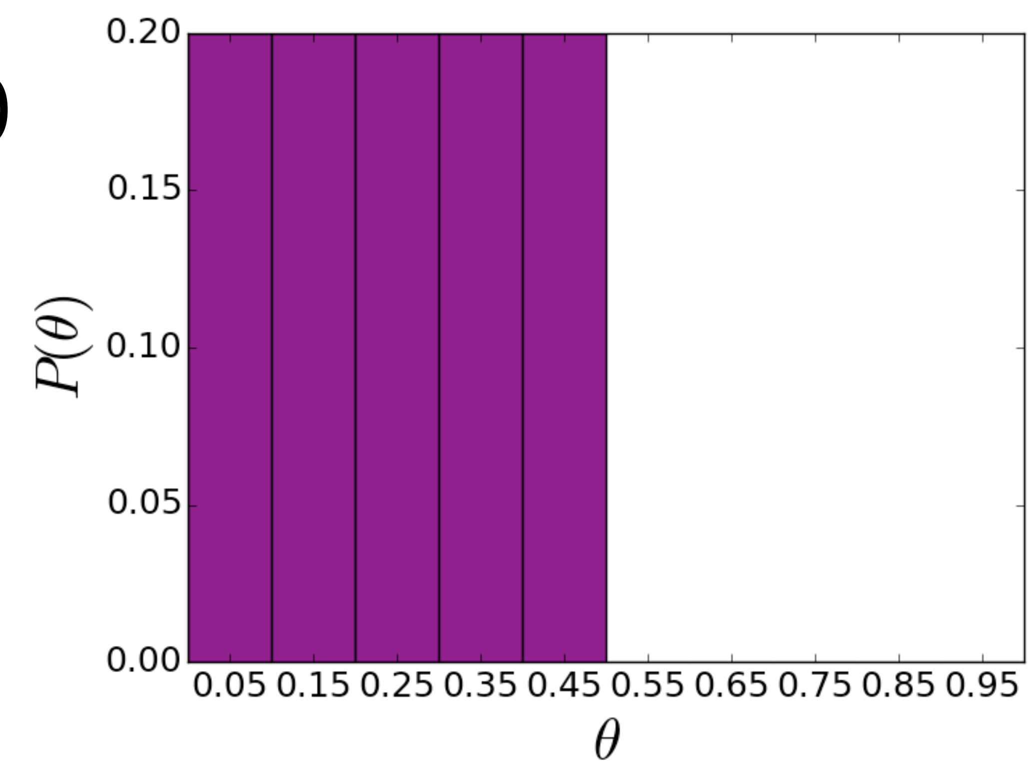
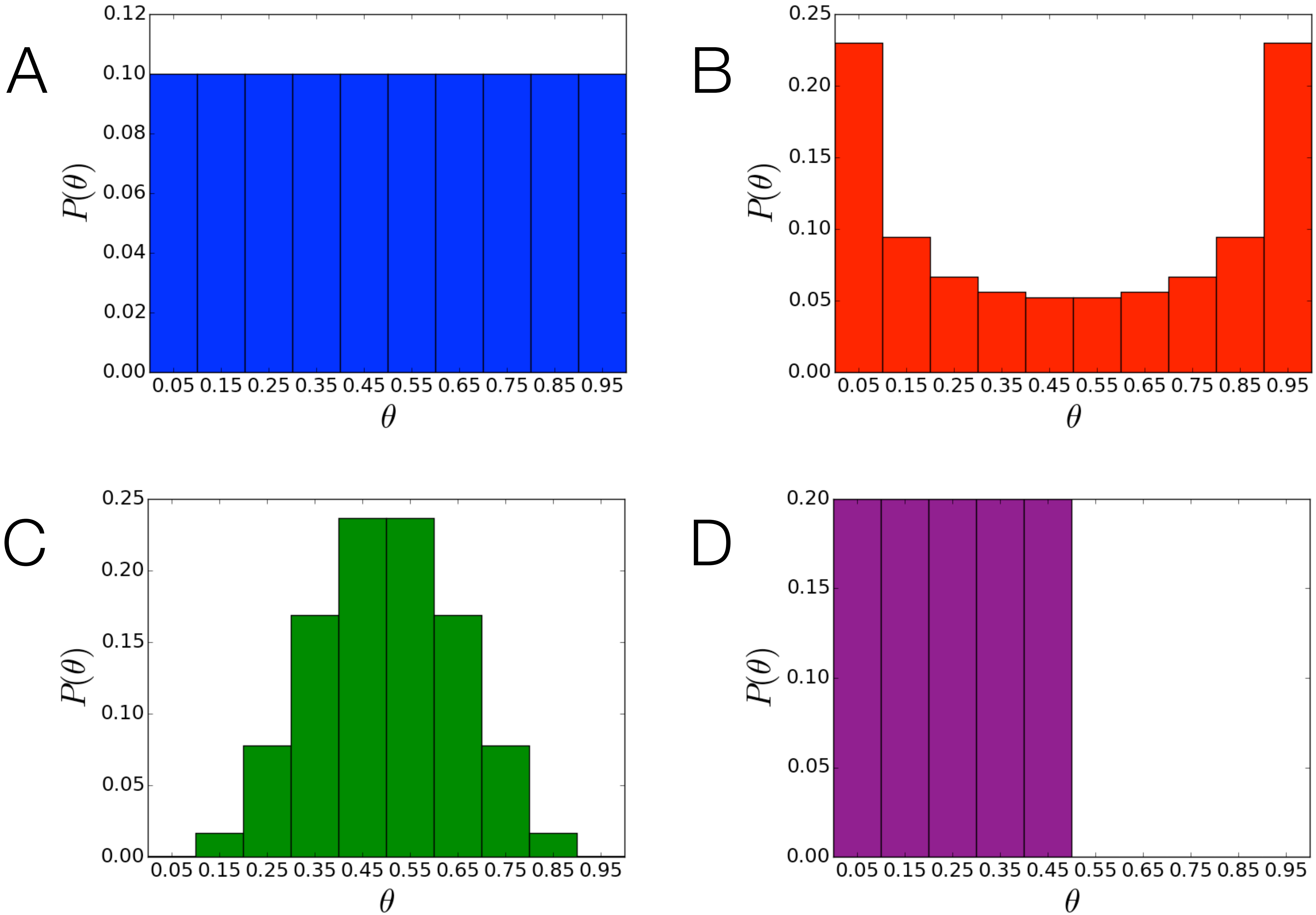


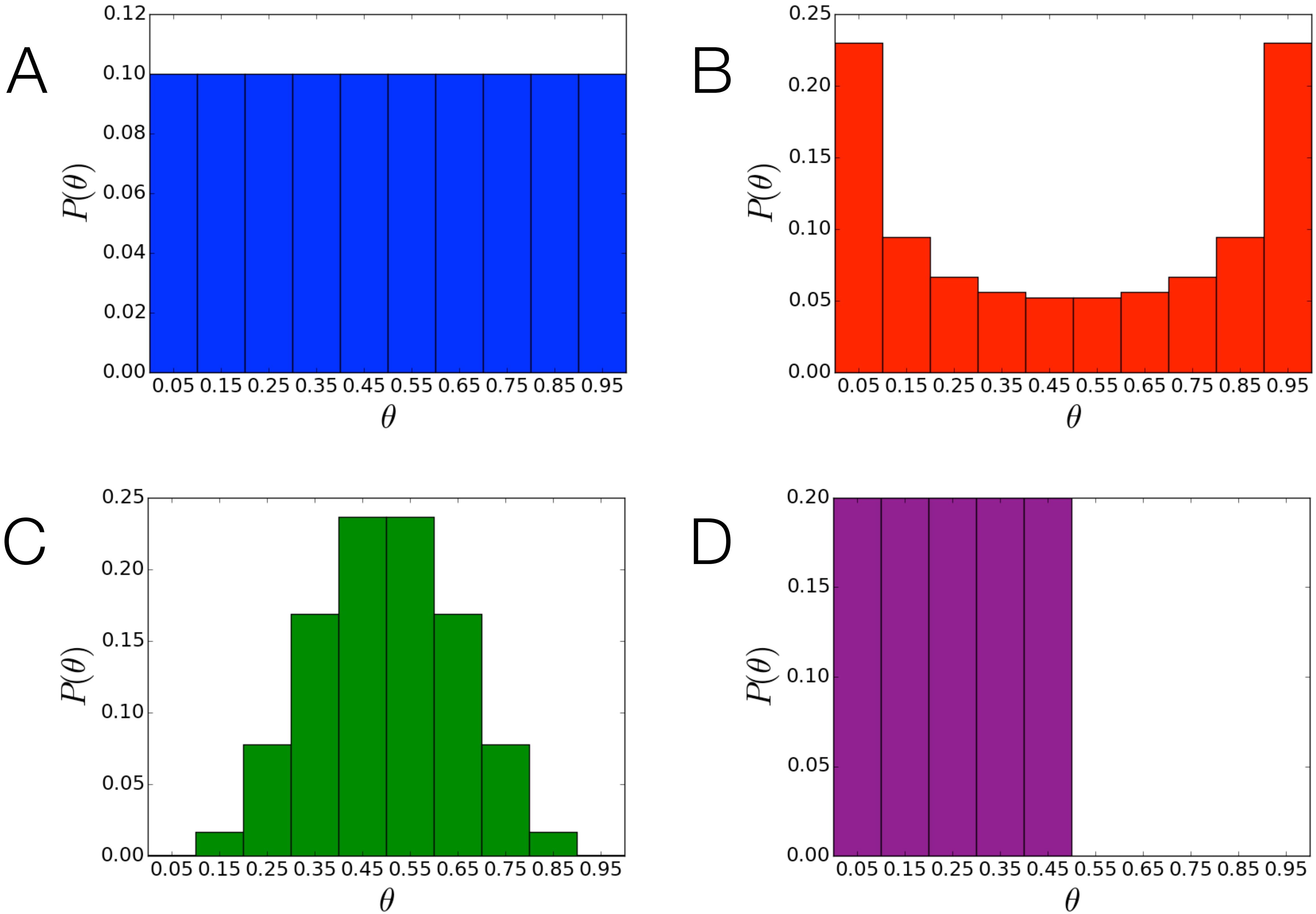
D



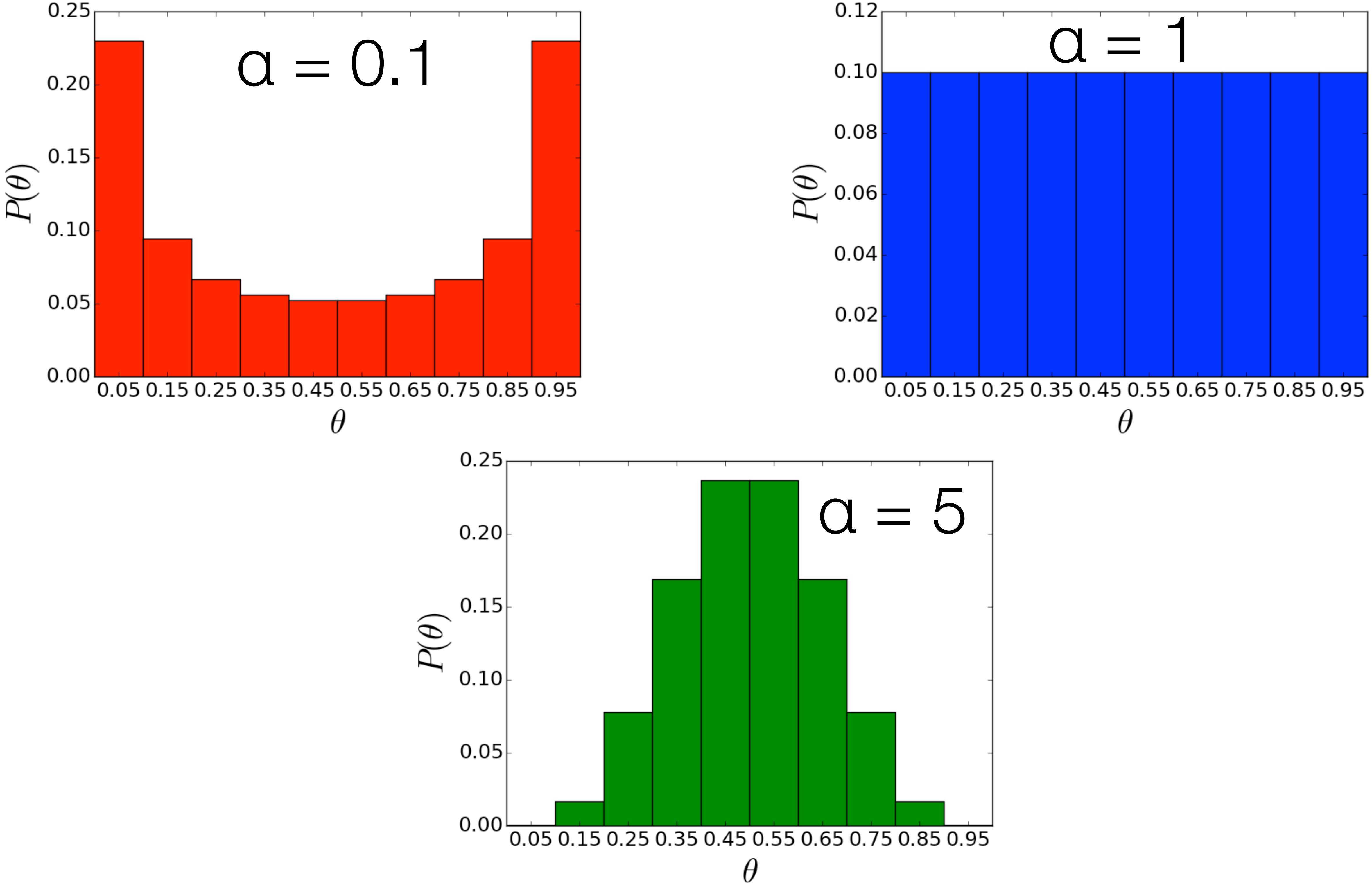
Which of these possible priors would be a good model for a **biased** learner, who thinks **each word should be used roughly equally often** (i.e. values of θ around 0.5 should be preferred)?



Which of these possible priors would be a good model for a **biased** learner, who thinks **only one word should be used** (i.e. values of θ close to 0 *or* close to 1 should be preferred)?

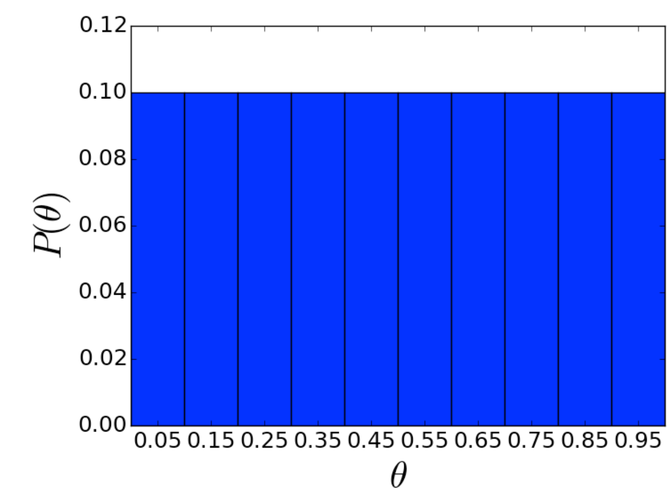


Our prior: the (symmetrical) beta distribution



Putting it together

- Let's say our learner considers 10 possible values of θ
  - 0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95
- They have a **uniform prior**



- And they have some data: d = [1,1]
- We can calculate the posterior probability for each possible value of θ
- This gives us a **posterior probability distribution**, and then we can just pick θ based on that (e.g. pick a value of θ according to its posterior probability)

$$P(\theta|d) \propto P(d|\theta)P(\theta)$$

Putting it together

$$P(\theta|d) \propto P(d|\theta)P(\theta)$$

- Uniform prior, d=[1,1]
- Consider just θ=0.25 and θ=0.75. Which has higher posterior probability?

A. P(θ = 0.25 | d) ≈ P(θ = 0.75 | d)

B. P(θ = 0.25 | d) is two times as big as P(θ = 0.75 | d)

C. P(θ = 0.25 | d) is nine times as big as P(θ = 0.75 | d)

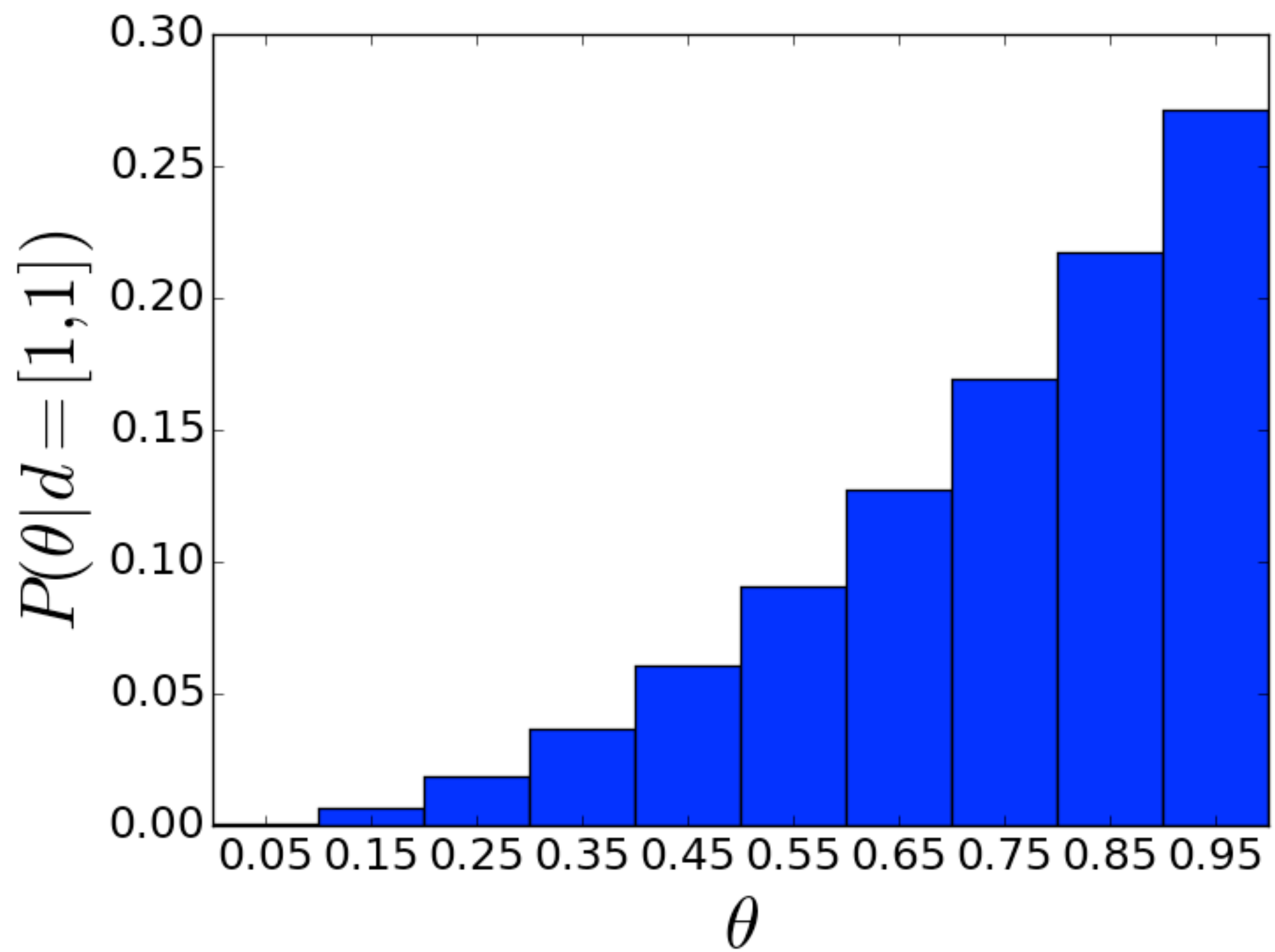
D. P(θ = 0.75 | d) is two as big as P(θ = 0.25 | d)

E. P(θ = 0.75 | d) is nine times as big as P(θ = 0.25 | d)

Putting it together

$$P(\theta|d) \propto P(d|\theta)P(\theta)$$

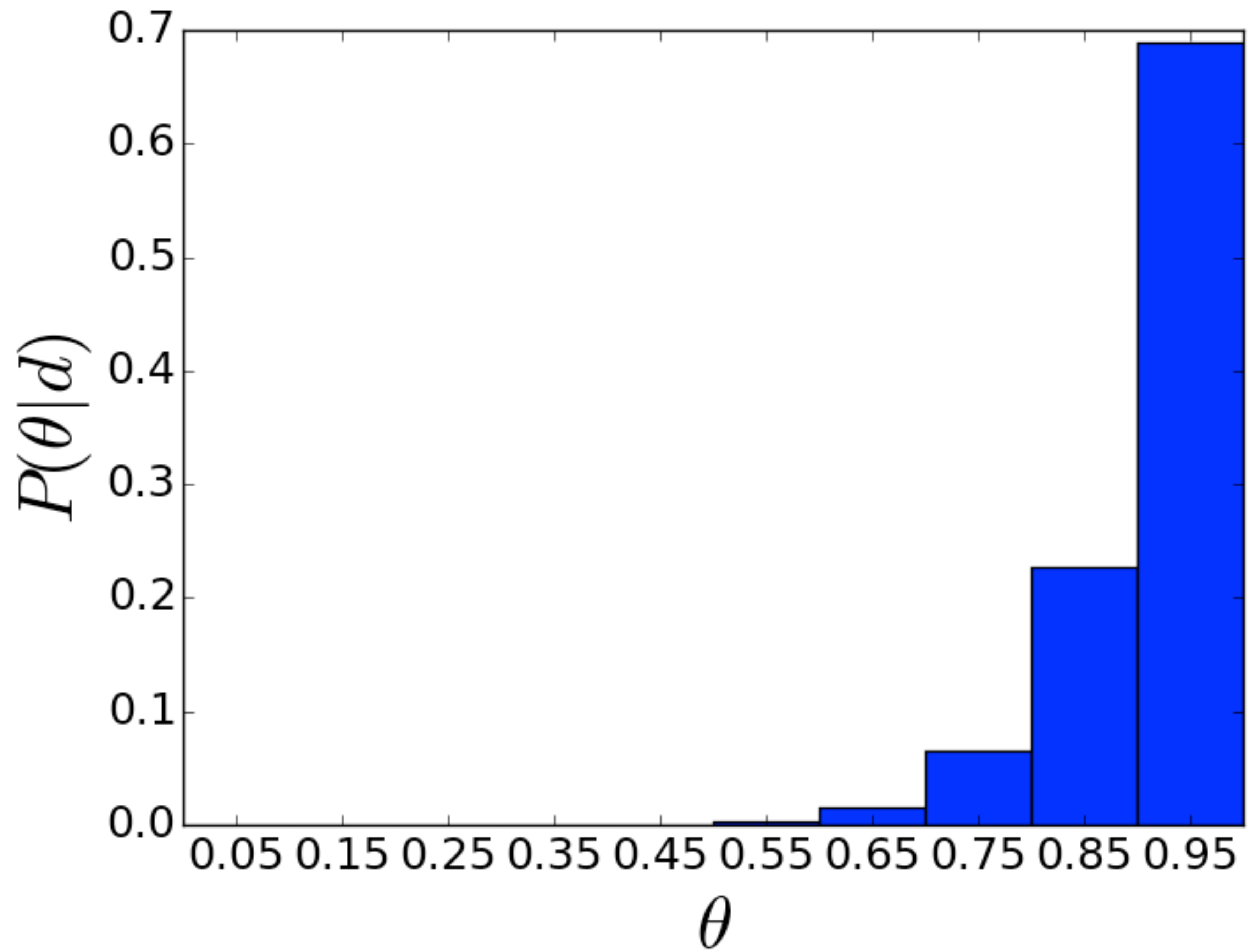
- Uniform prior, d=[1,1]



Putting it together

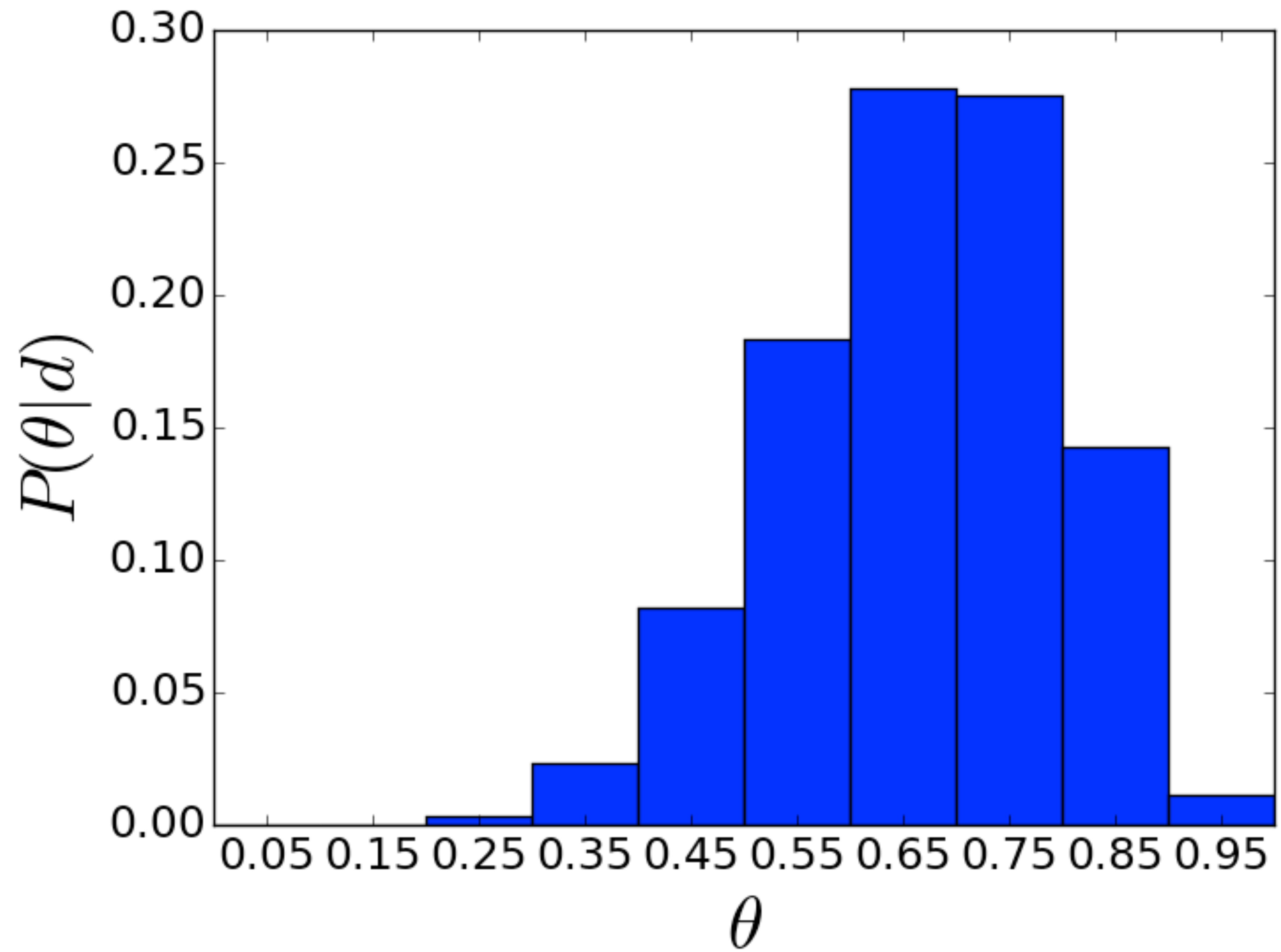
$$P(\theta|d) \propto P(d|\theta)P(\theta)$$

- Uniform prior, d=[1,1,1,1,1,1,1,1,1,1]



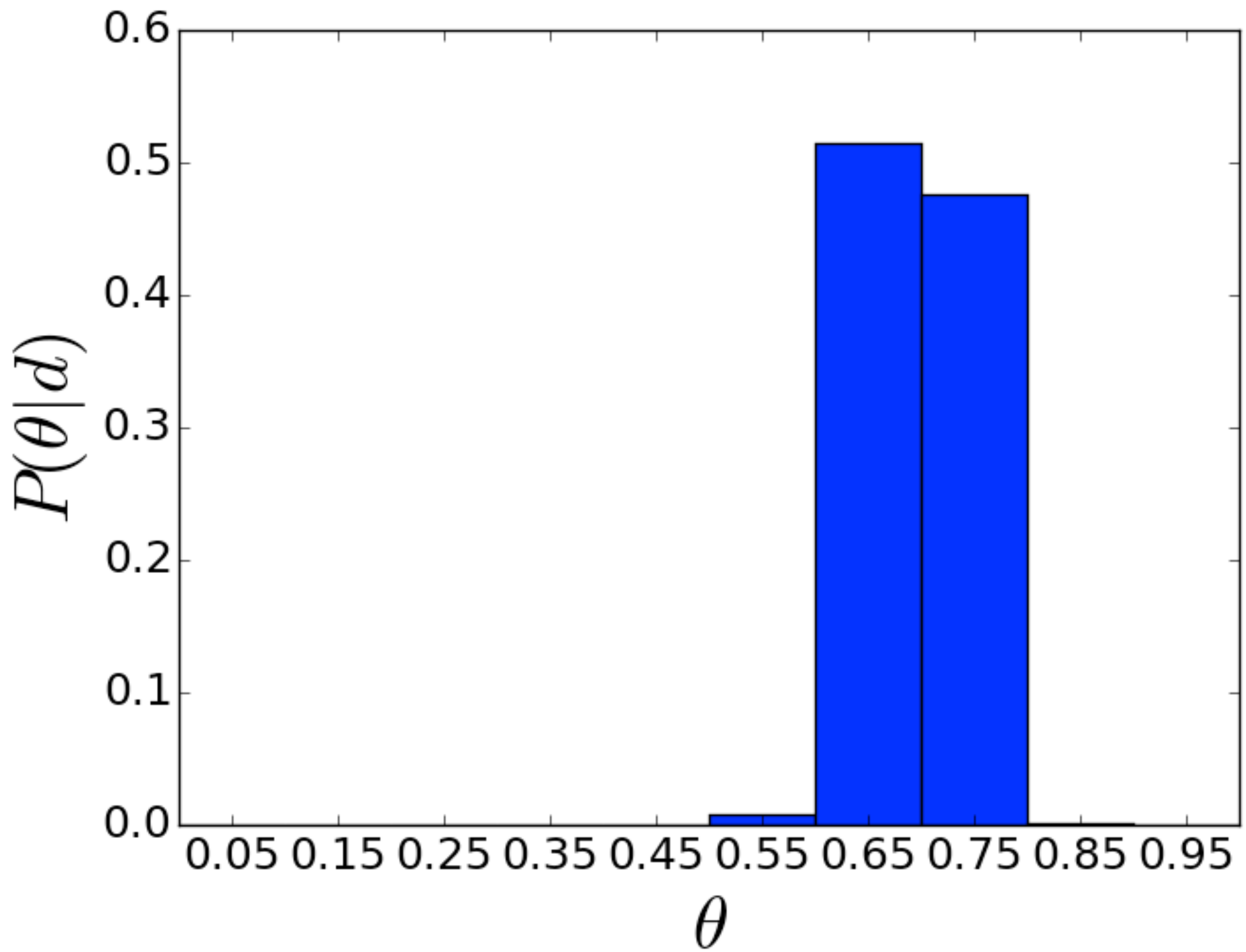
Putting it together $P(\theta|d) \propto P(d|\theta)P(\theta)$

- Uniform prior, d=[1,1,1,1,1,1,1,0,0,0]



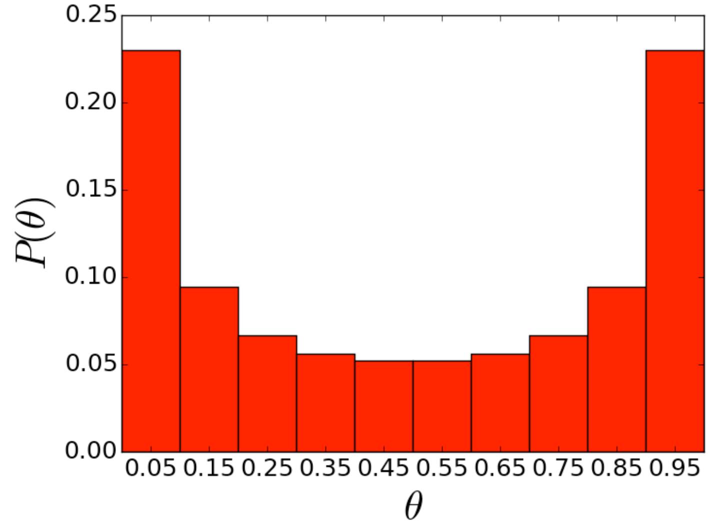
Putting it together $P(\theta|d) \propto P(d|\theta)P(\theta)$

- Uniform prior, d=[70 occurrences of word 1, 30 of word 0]



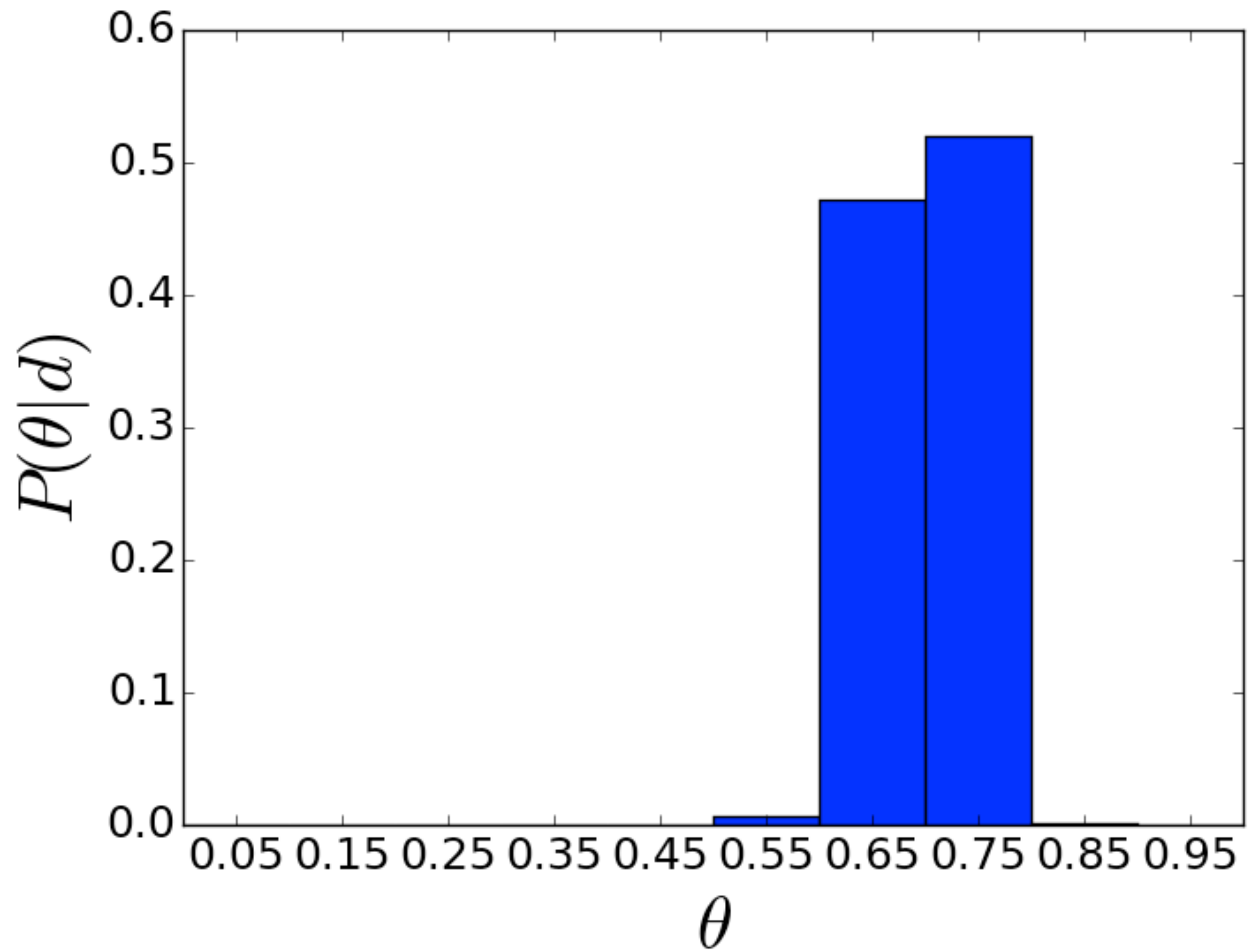
Putting it together $P(\theta|d) \propto P(d|\theta)P(\theta)$

- Regularity prior, d= [70 occurrences of word 1, 30 of word 0]



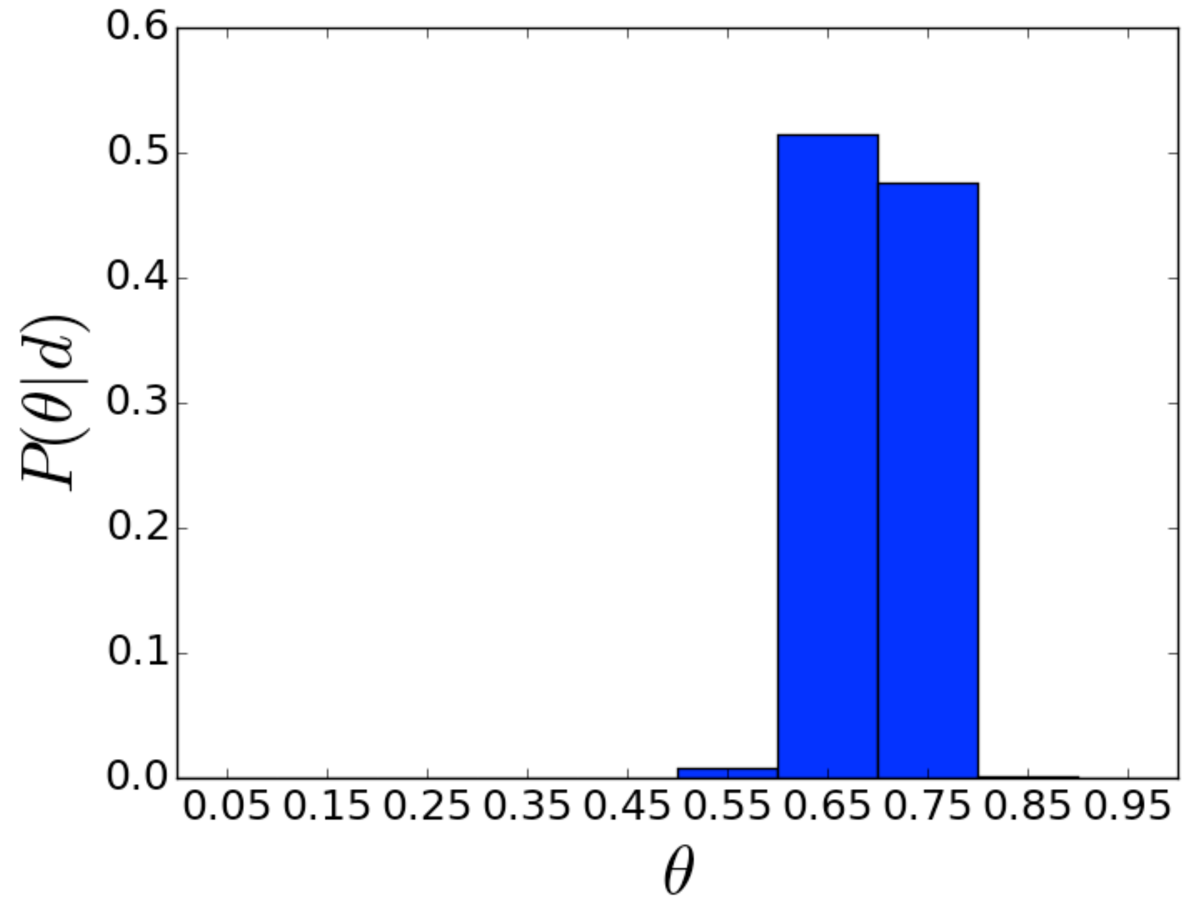
Putting it together $P(\theta|d) \propto P(d|\theta)P(\theta)$

- Regularity prior, d= [70 occurrences of word 1, 30 of word 0]

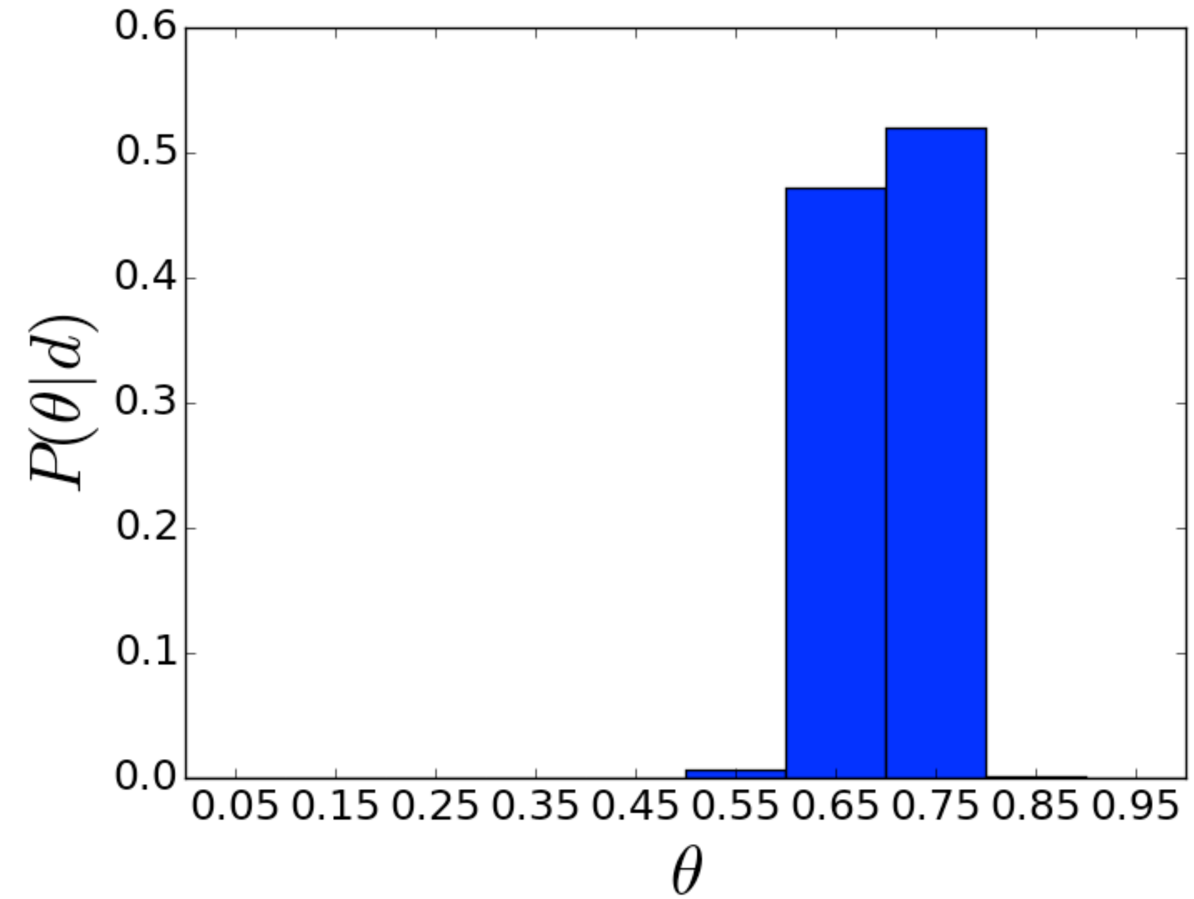


Data obscures the prior $P(\theta|d) \propto P(d|\theta)P(\theta)$

Unbiased learner



Biased learner



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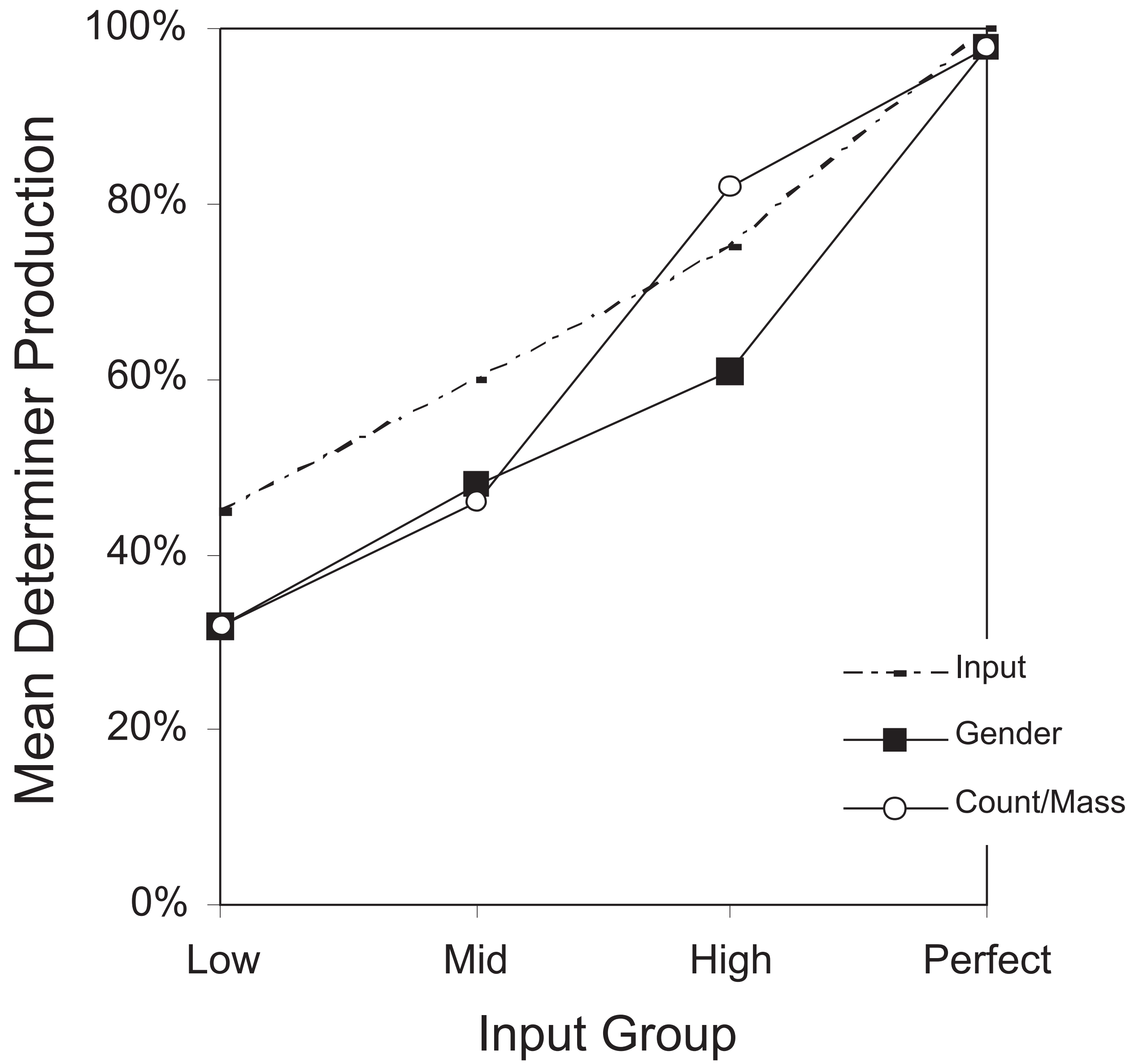
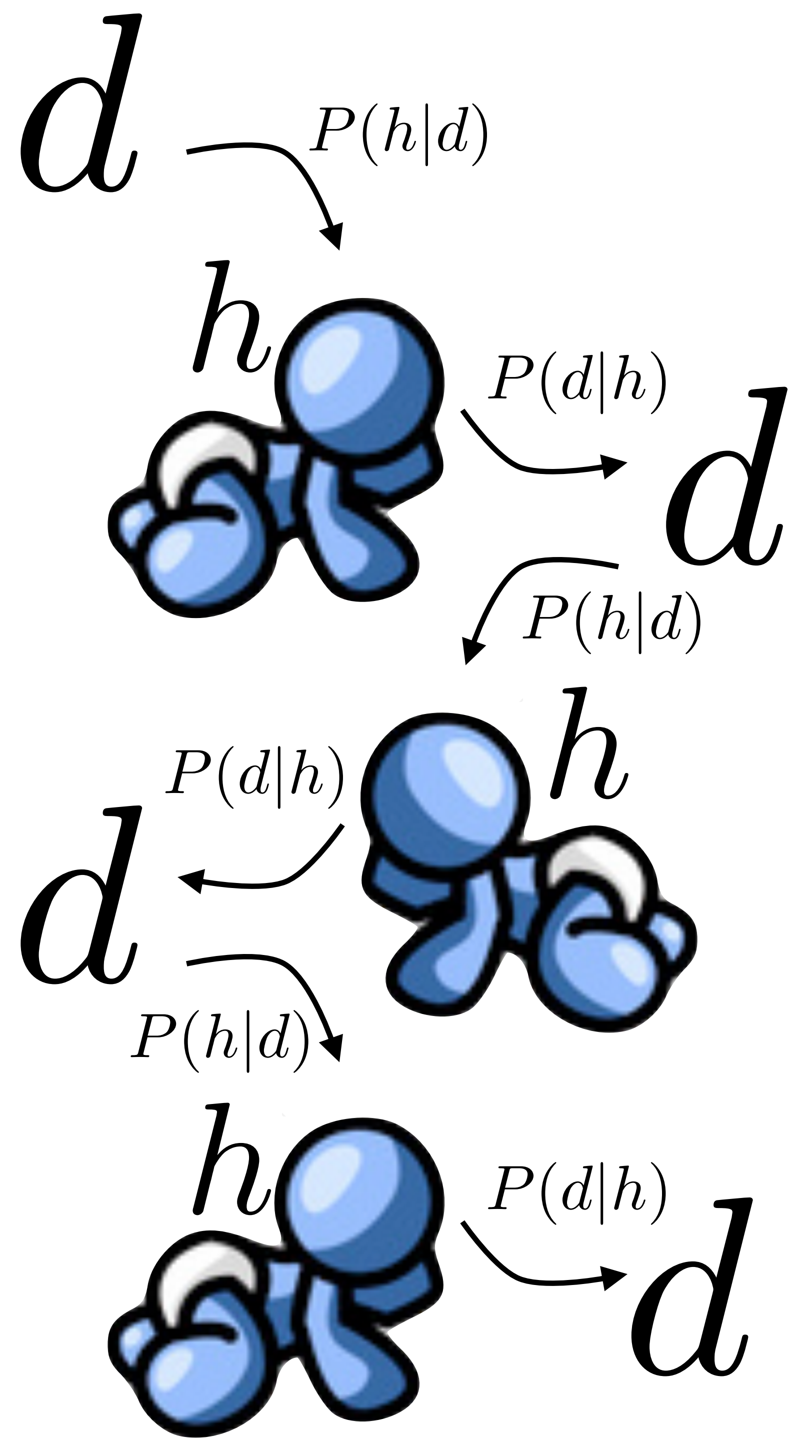
Unbiased learner? Biased learner?

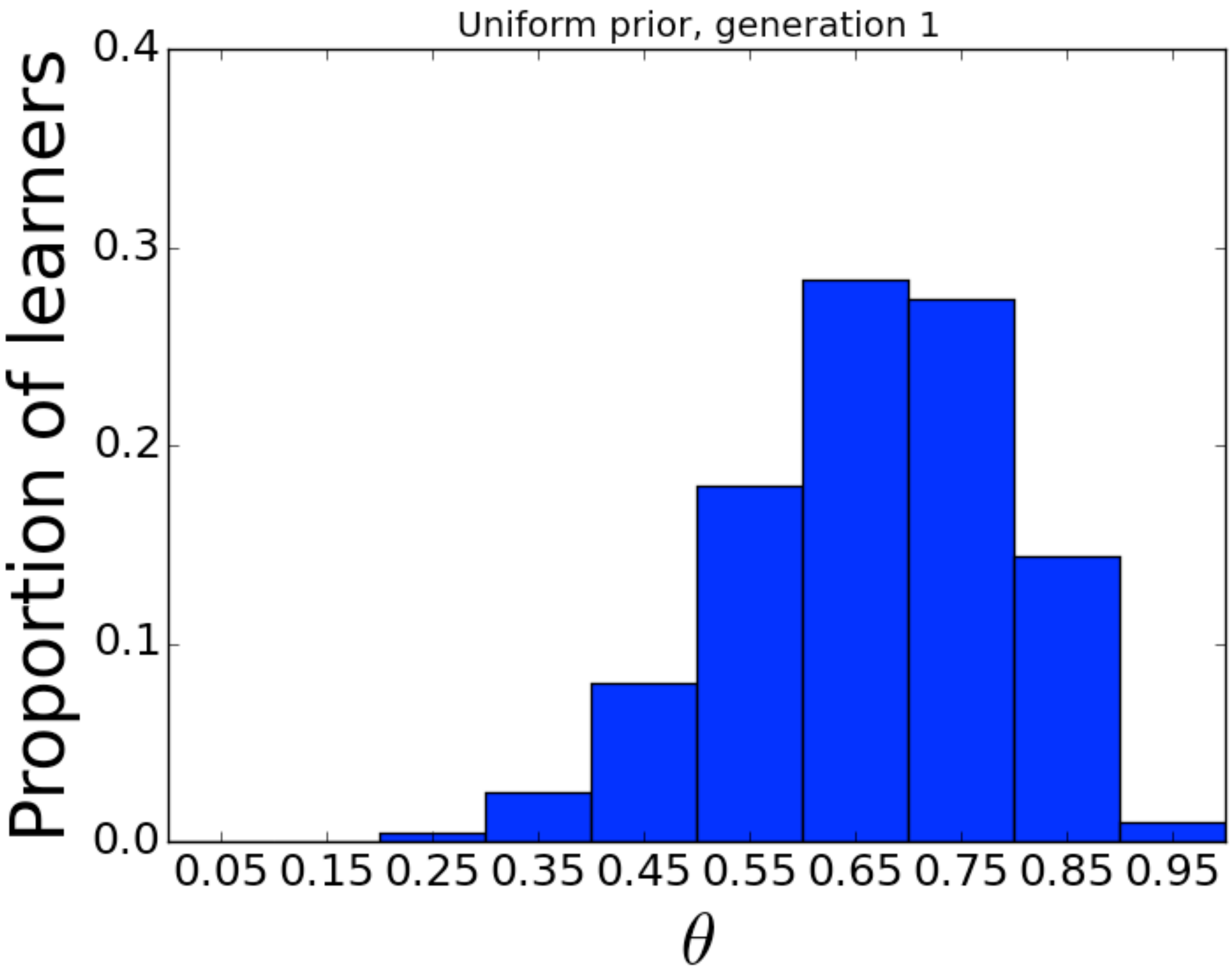


The solution: iterated learning



Over time, the bias will reveal itself?

Watching the prior reveal itself



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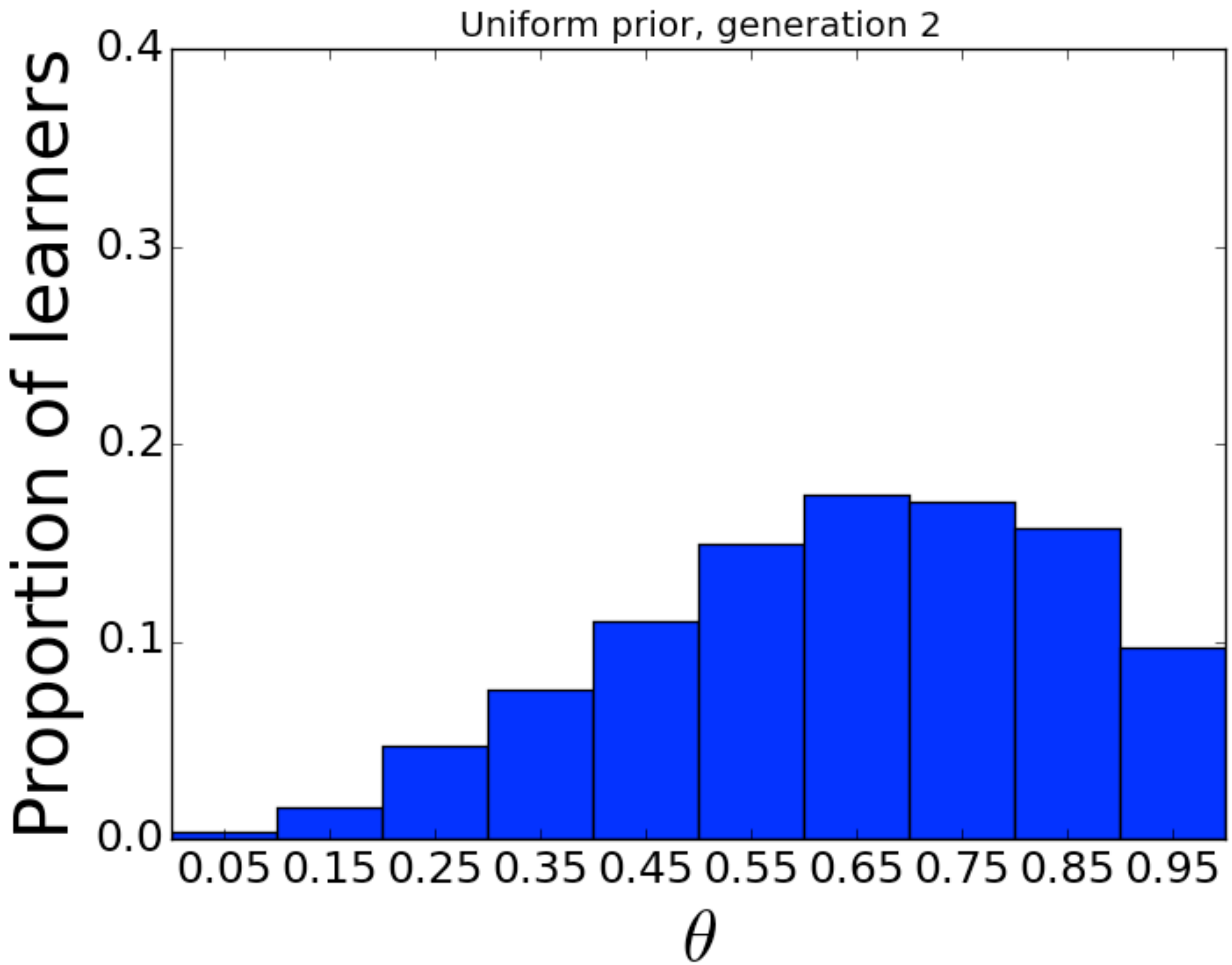

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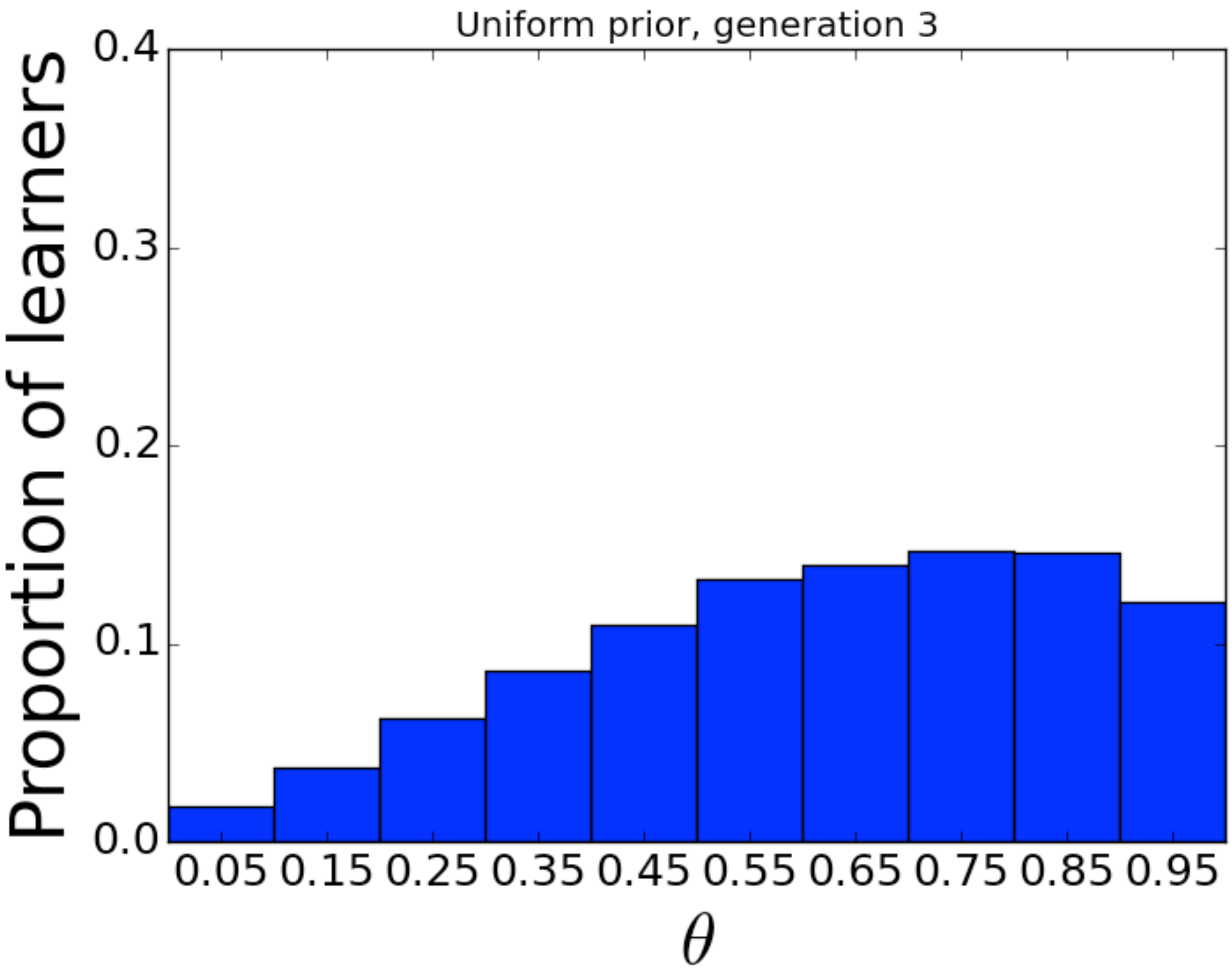

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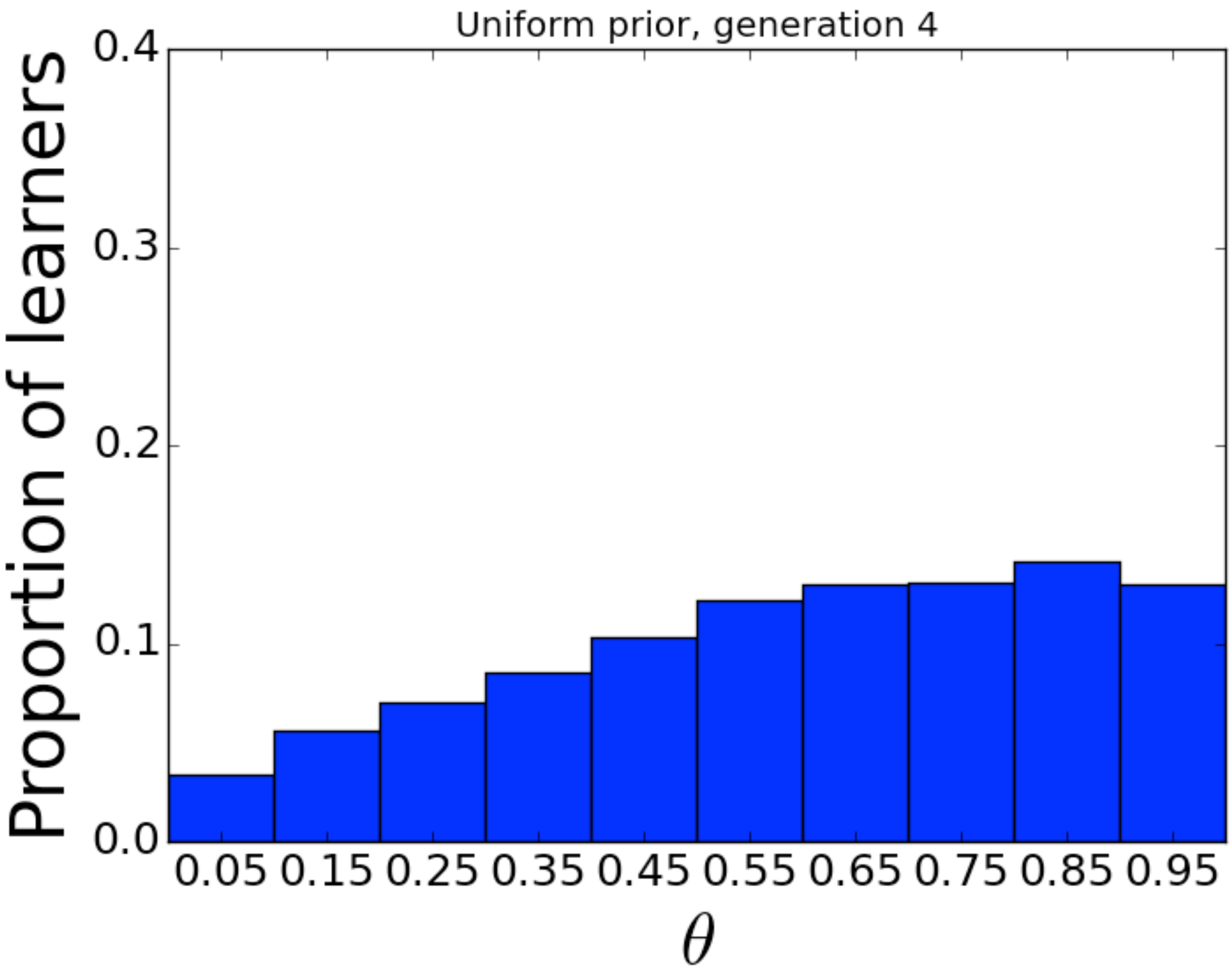

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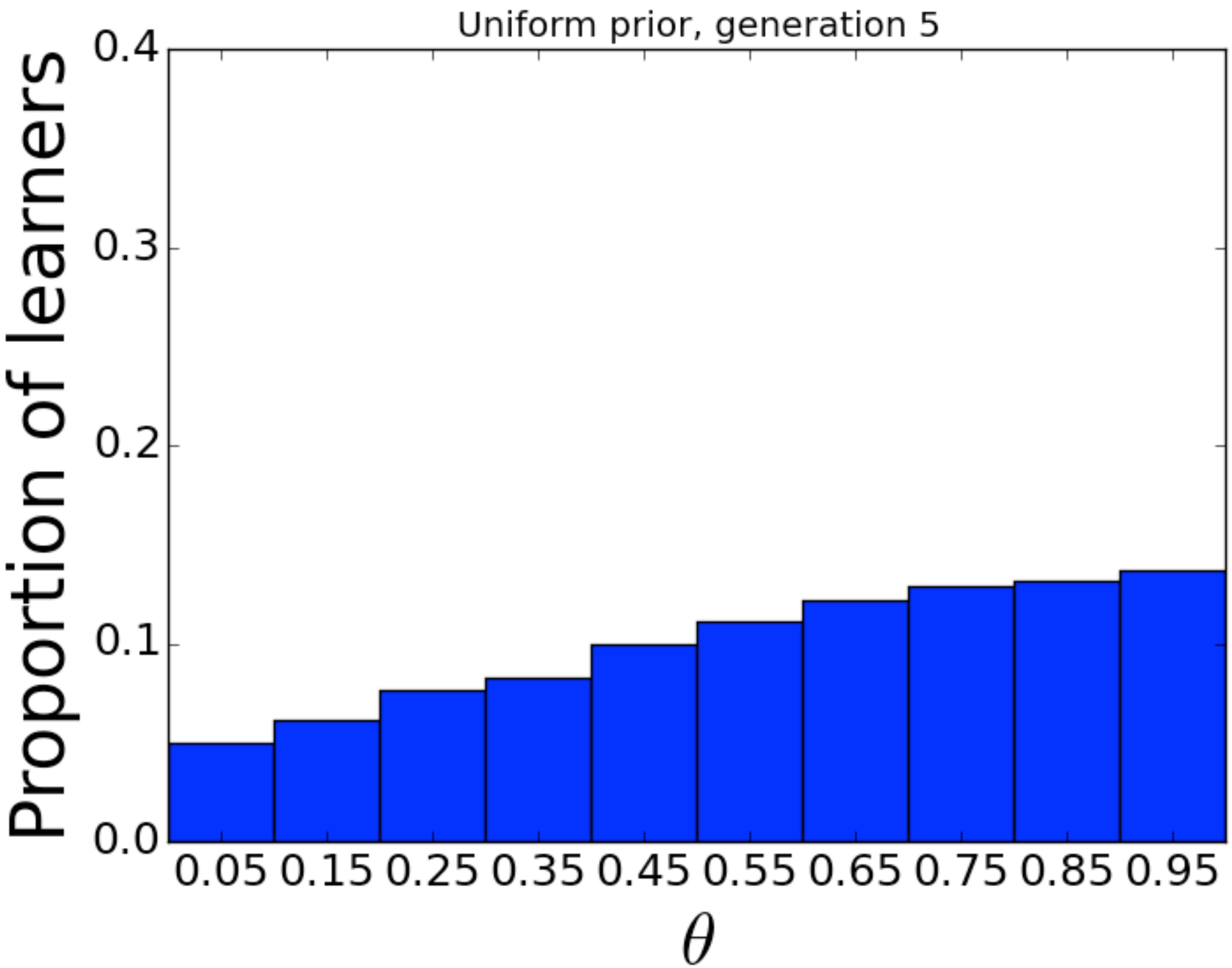

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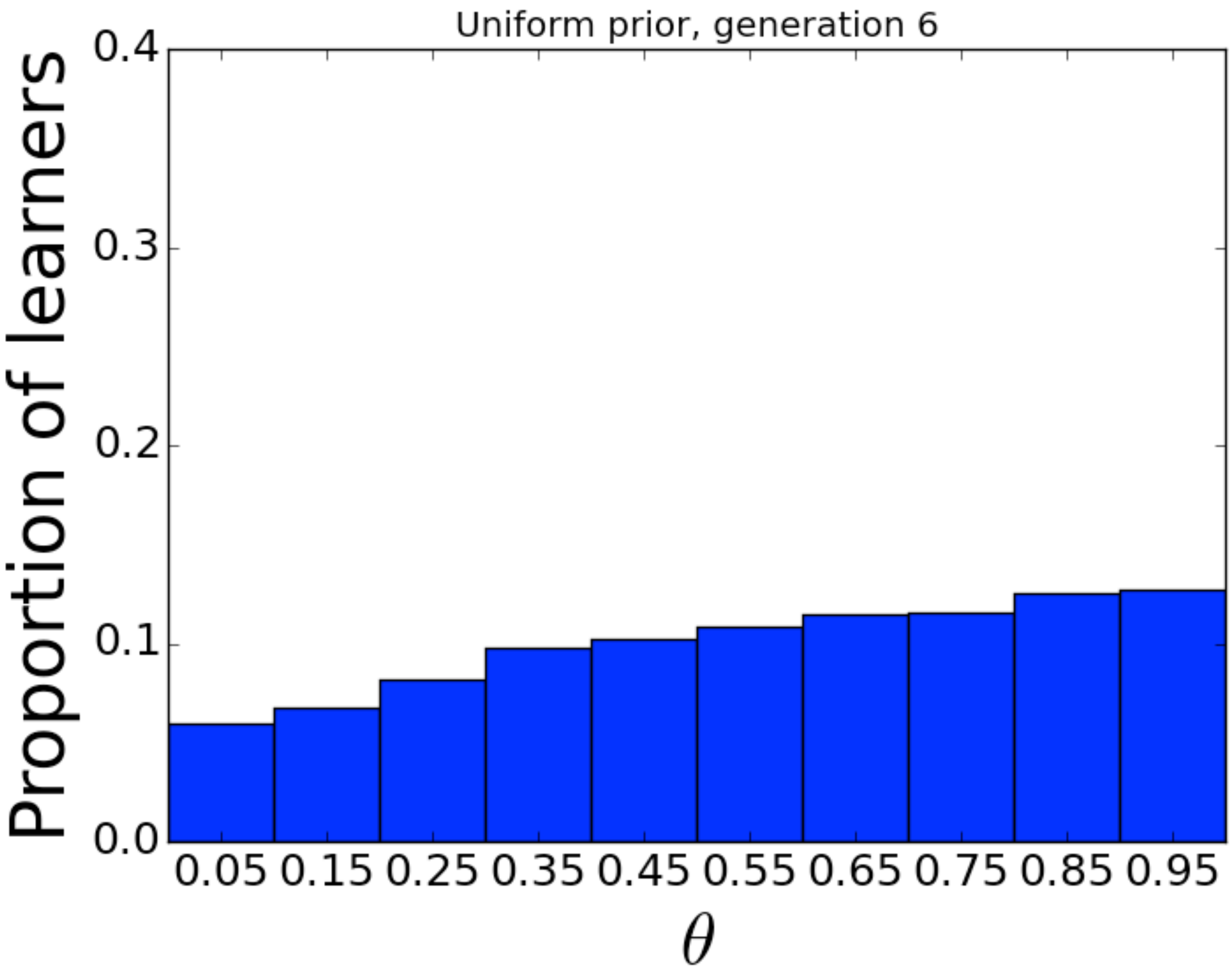

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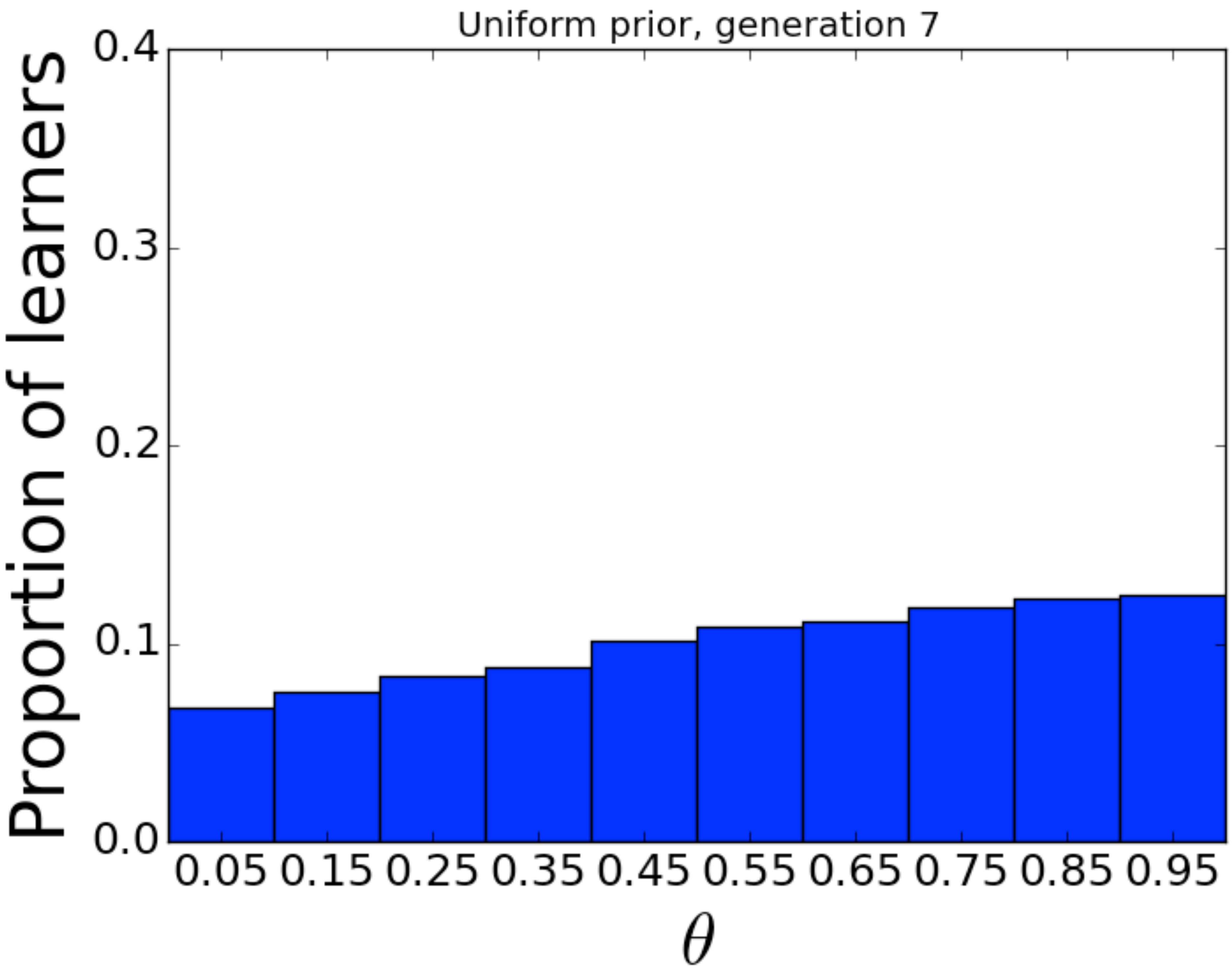

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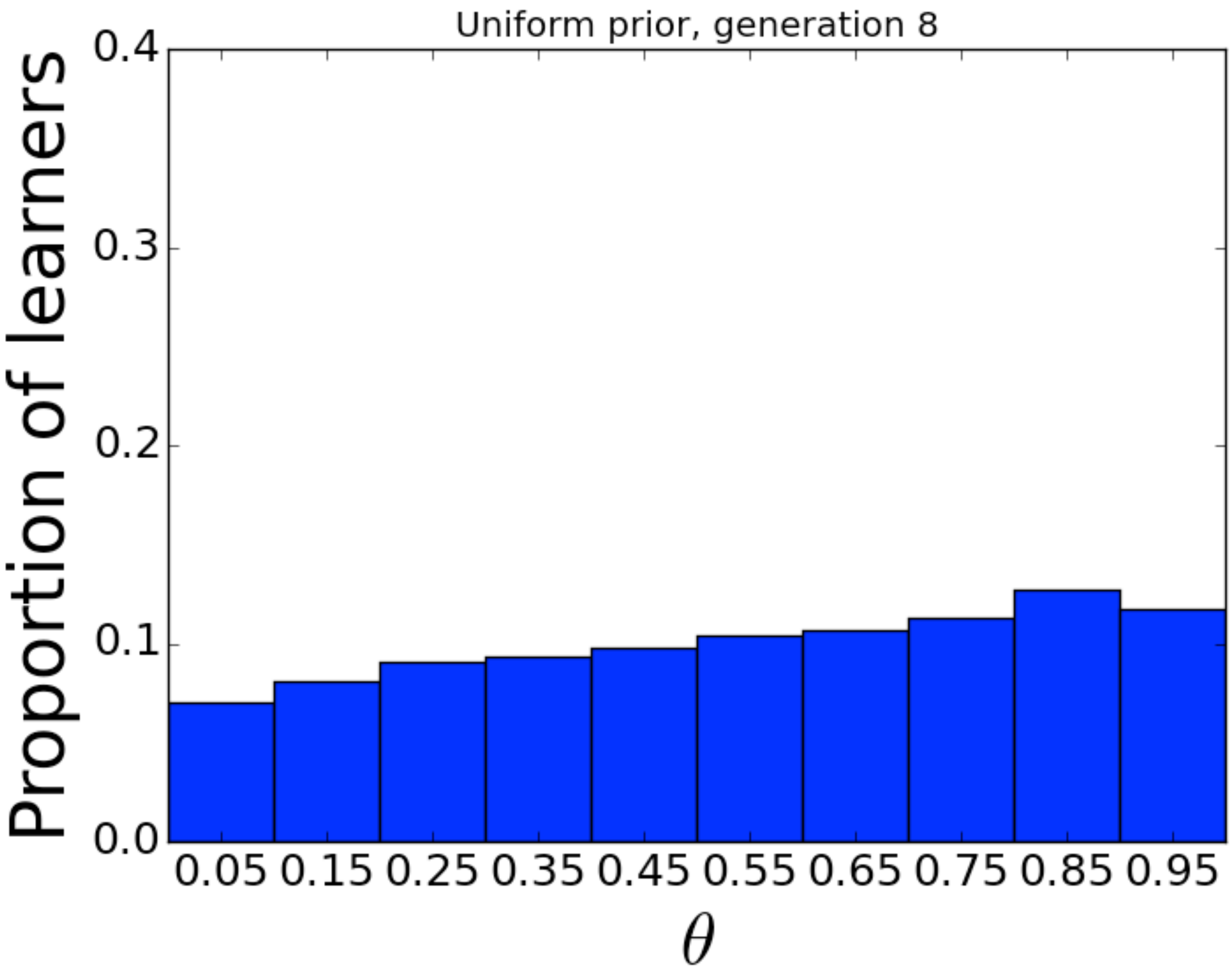

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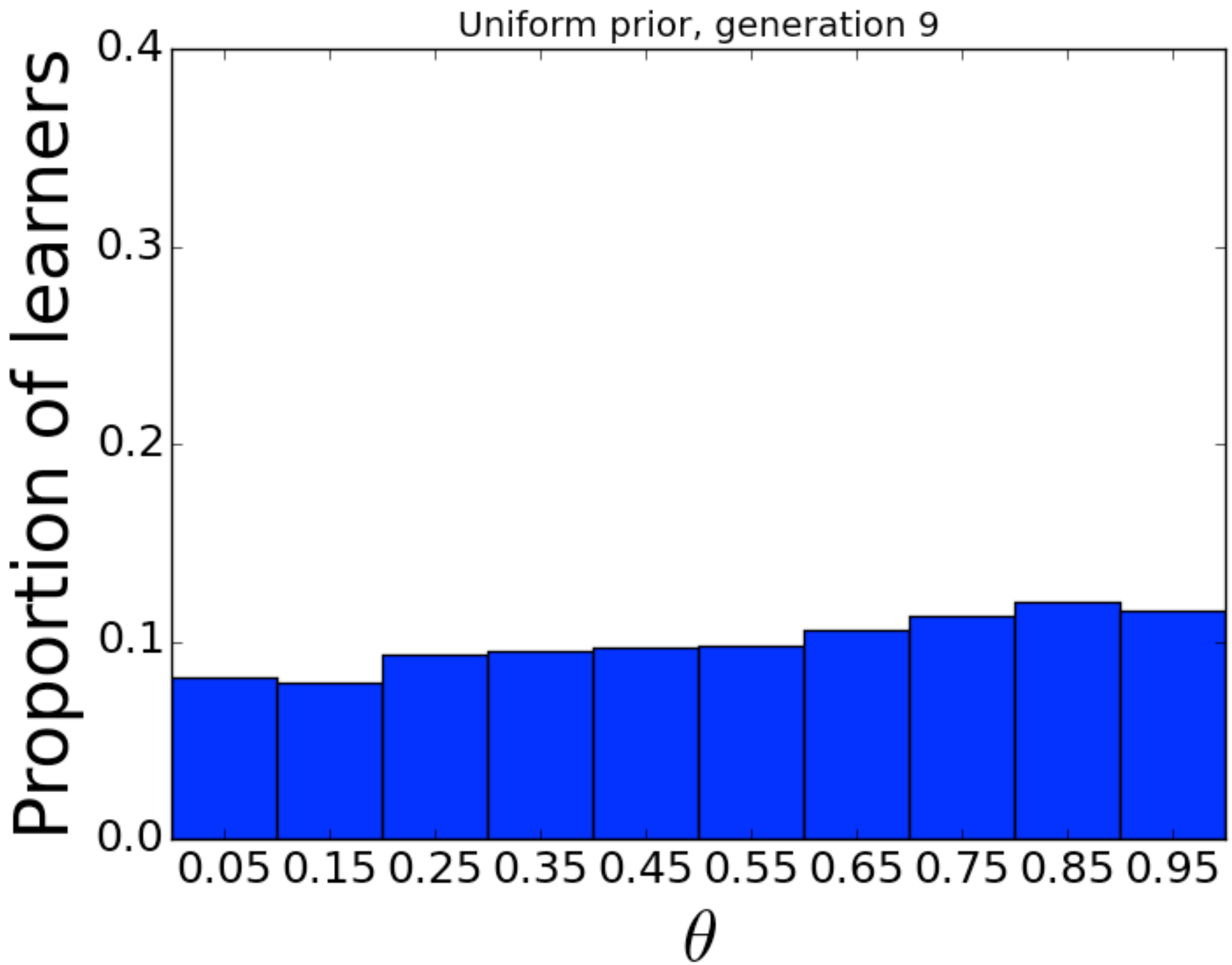

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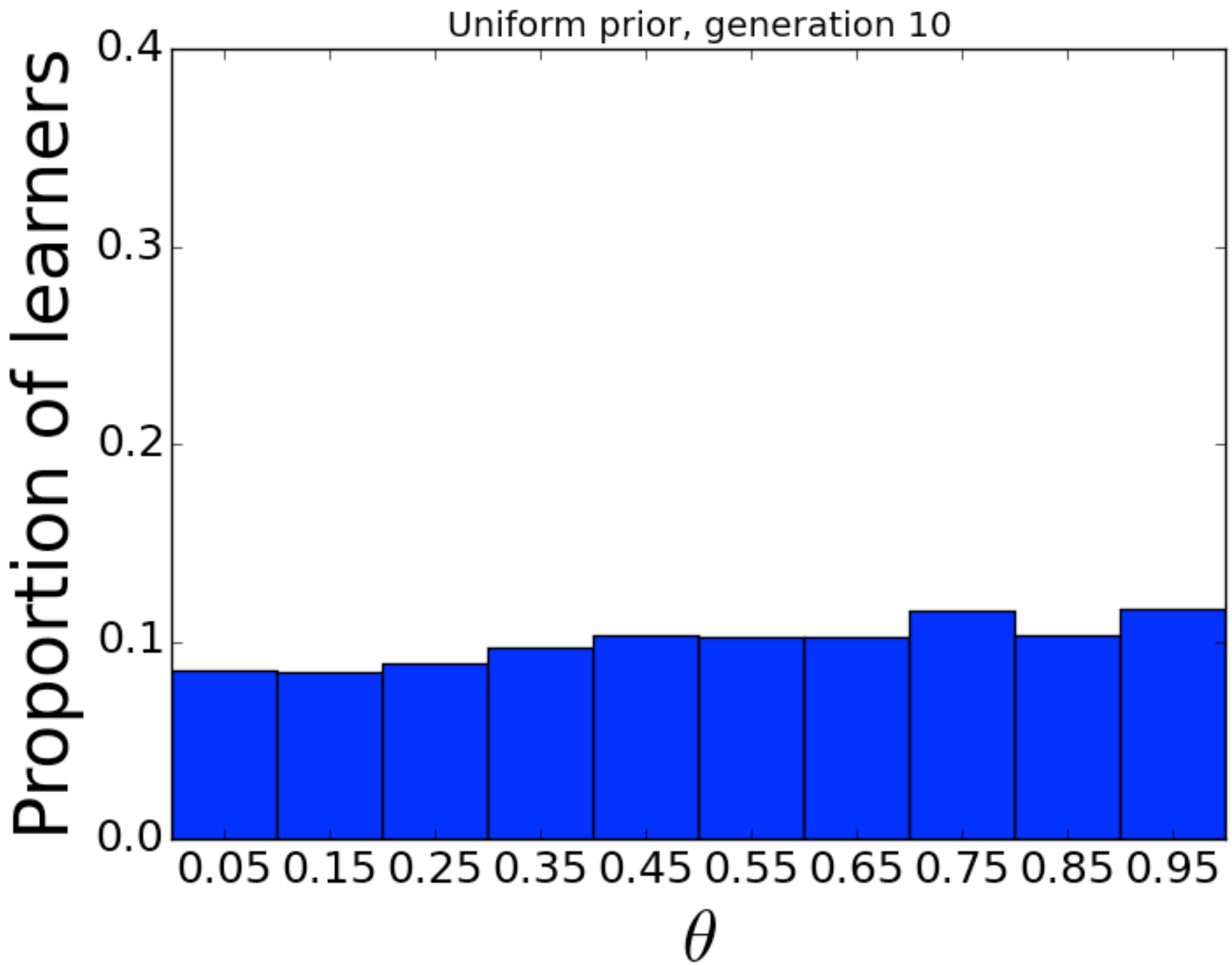

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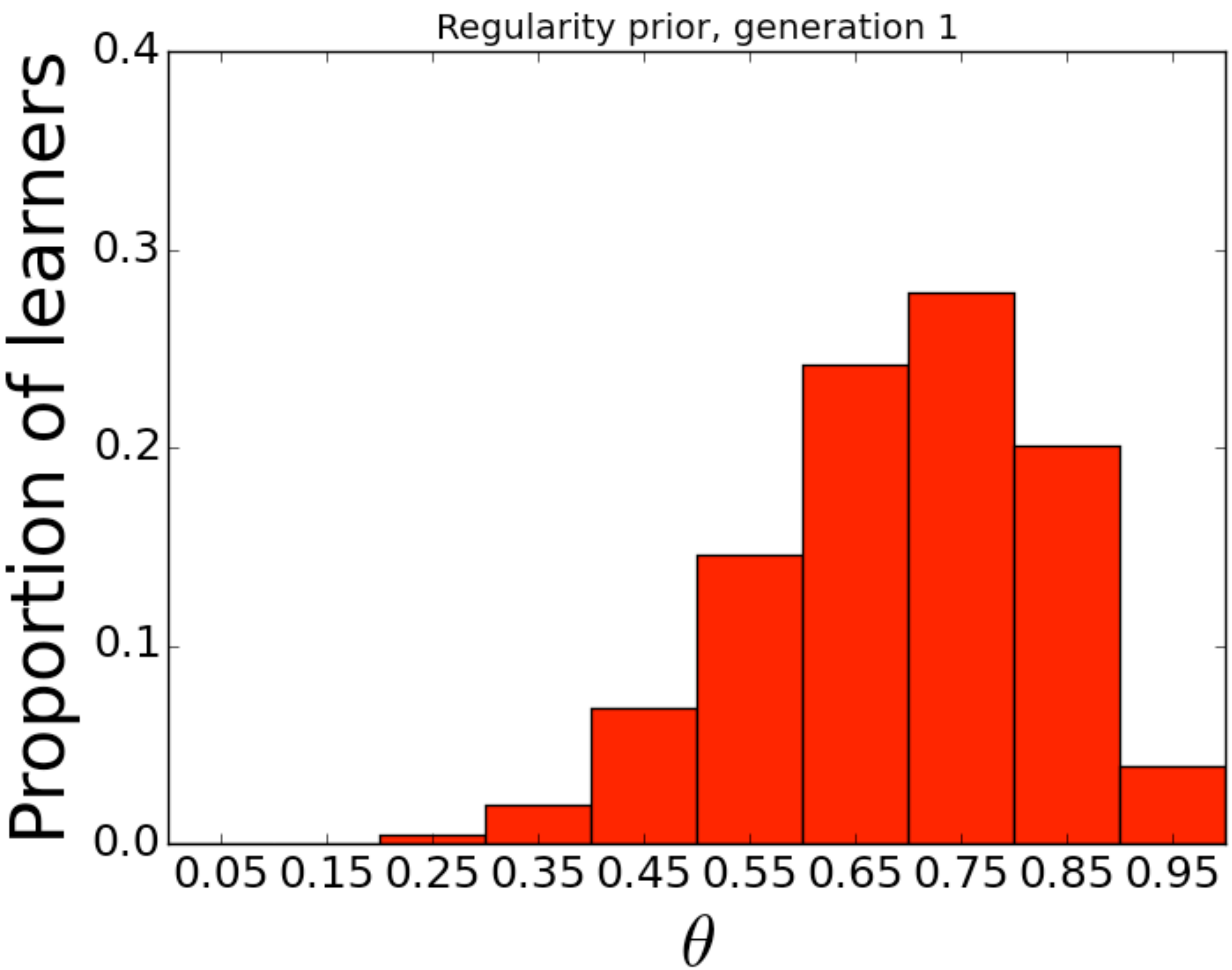

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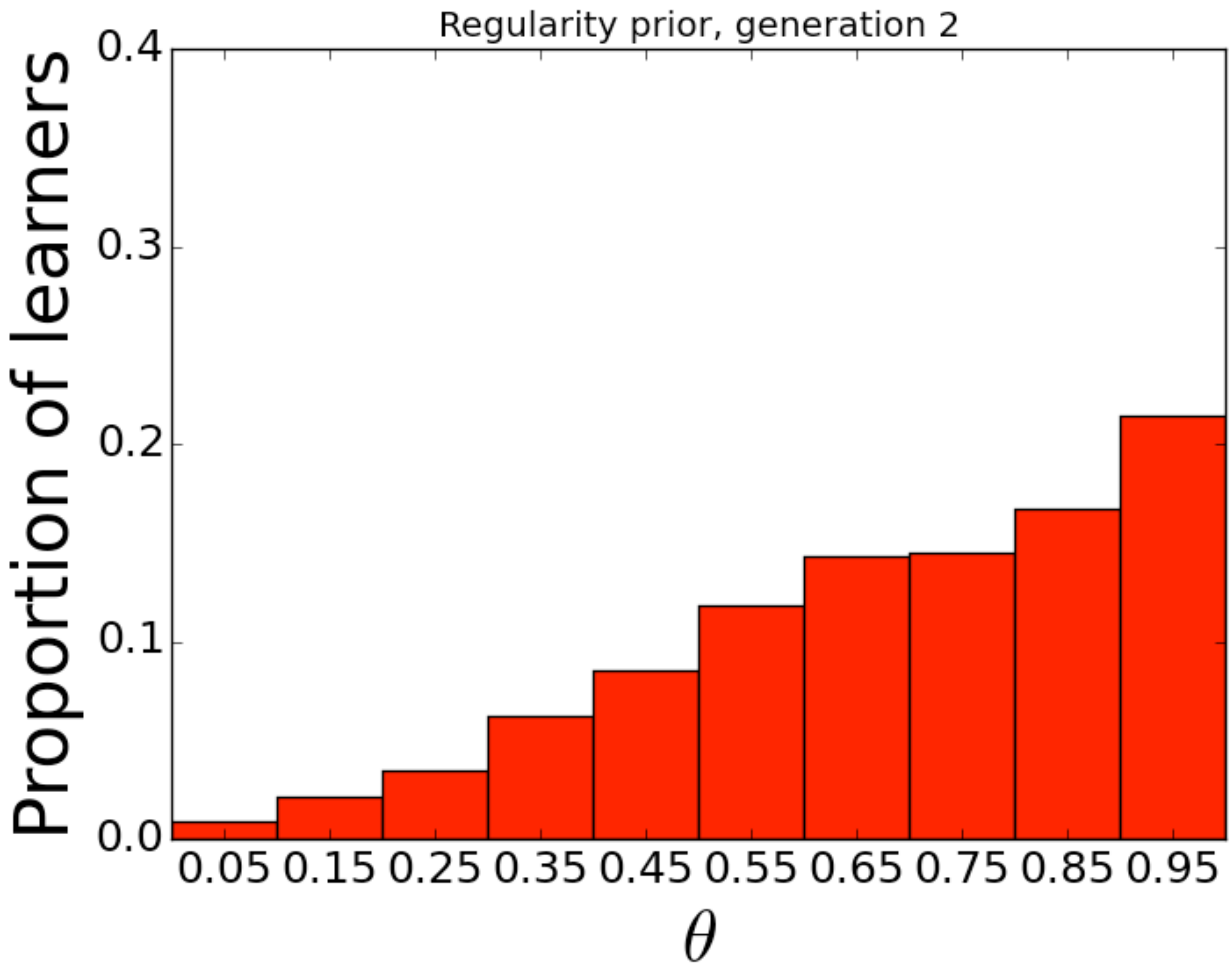

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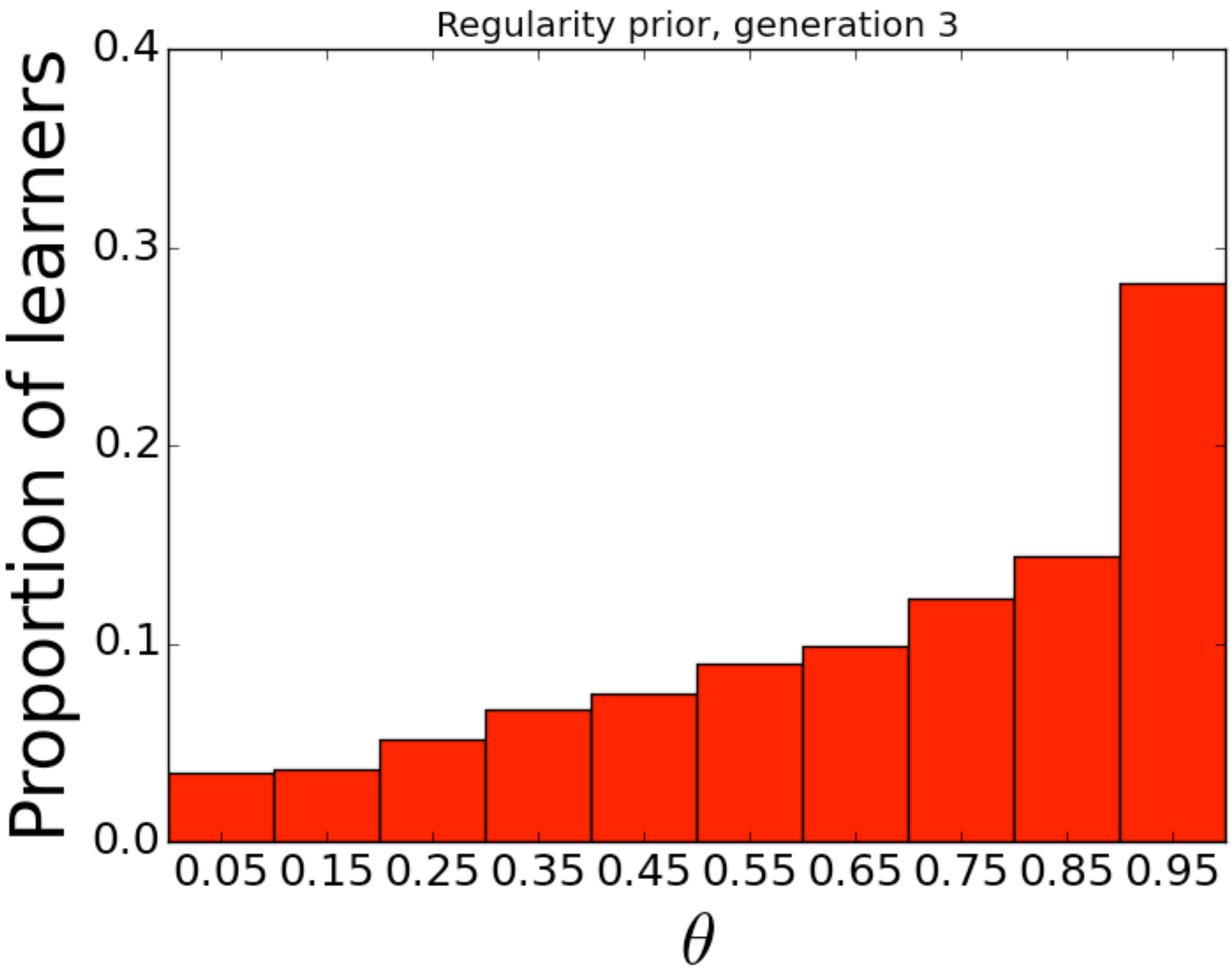

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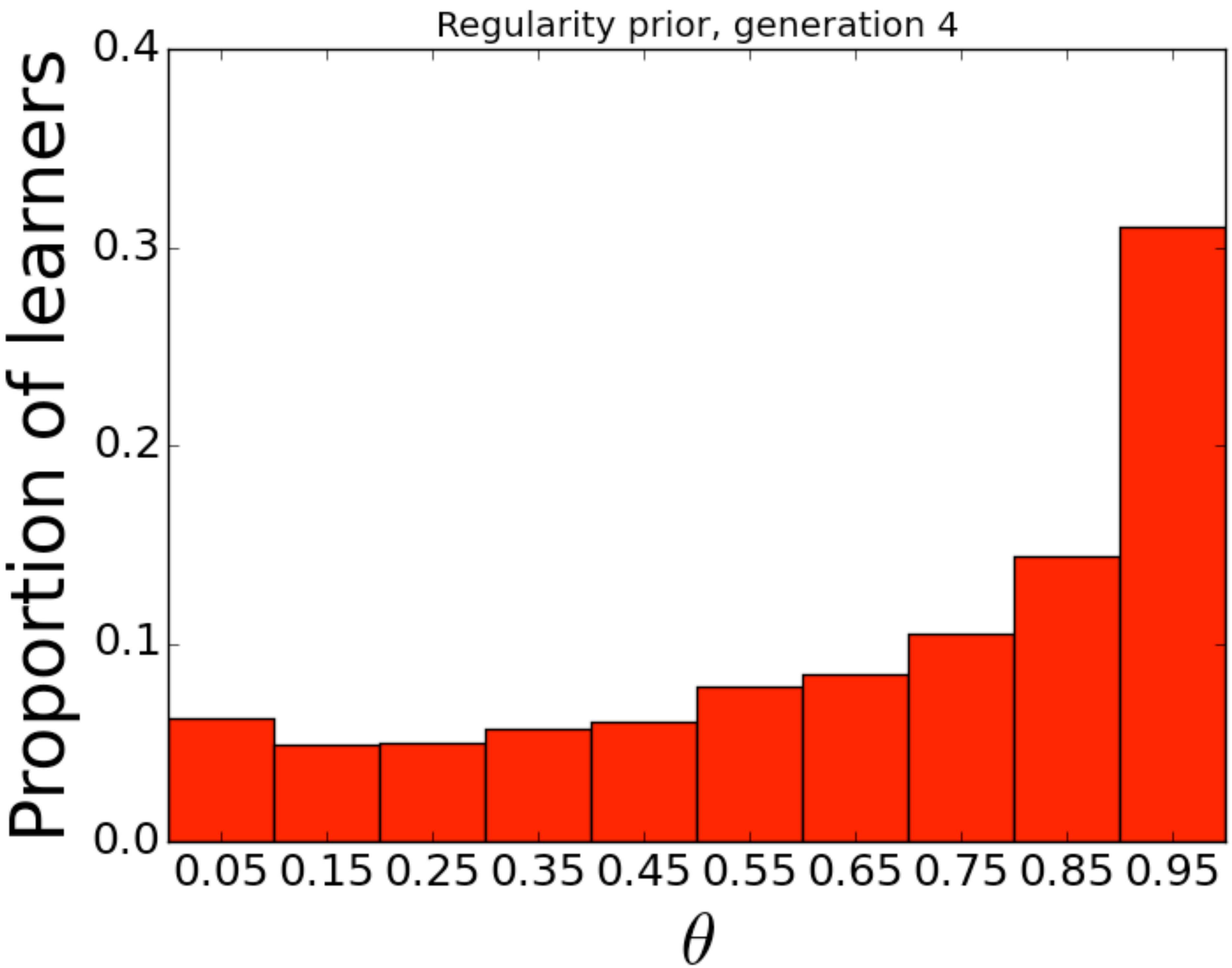

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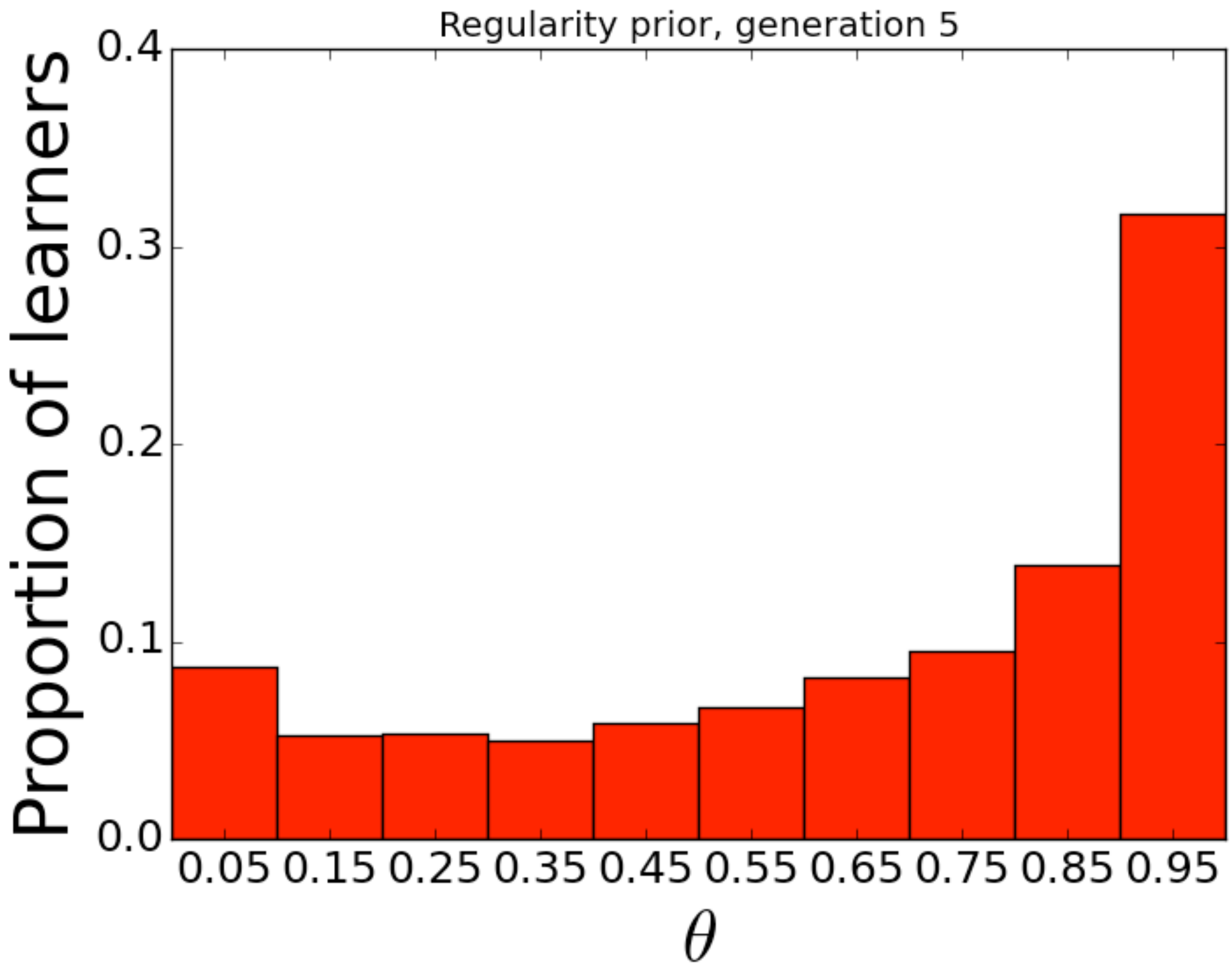

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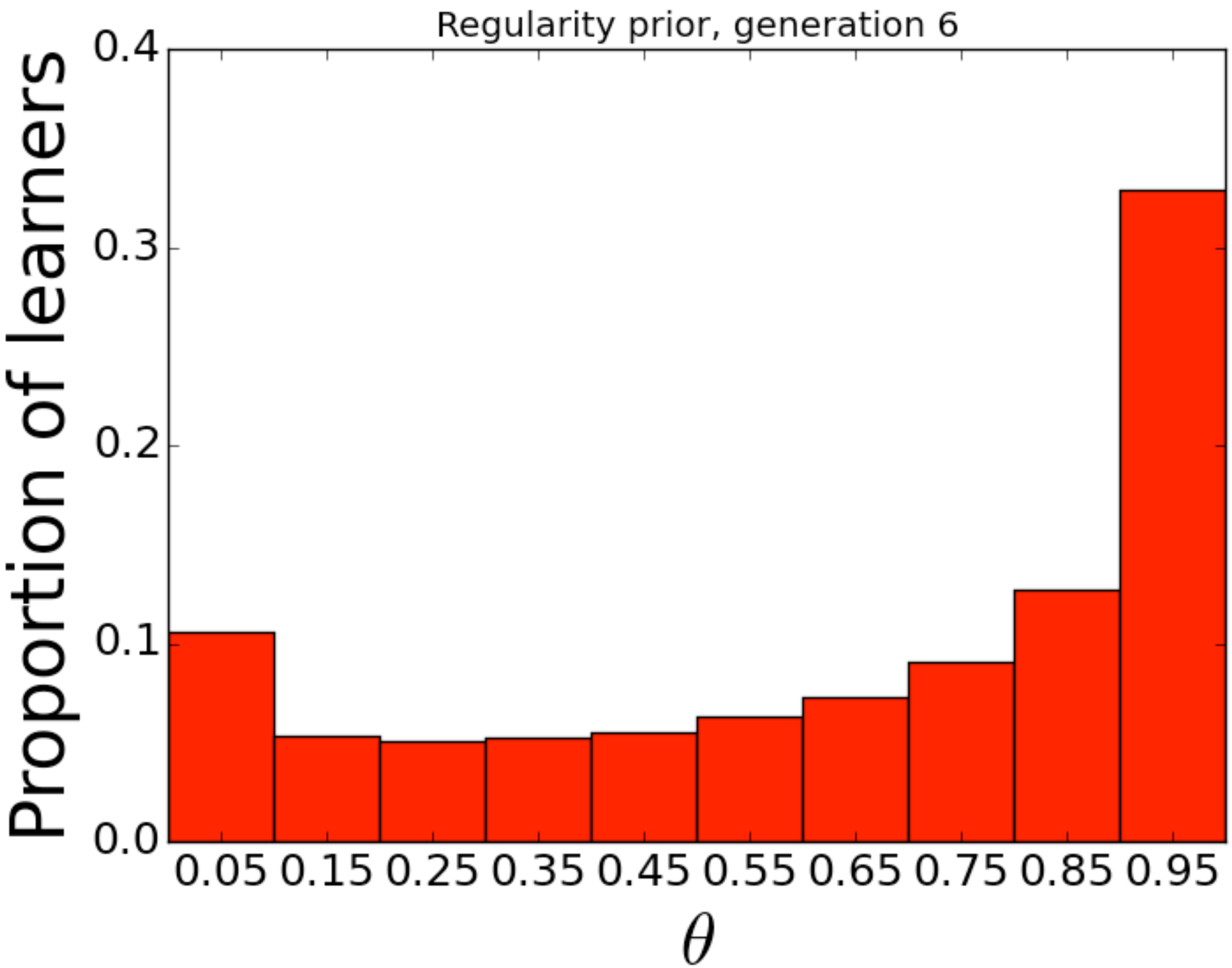

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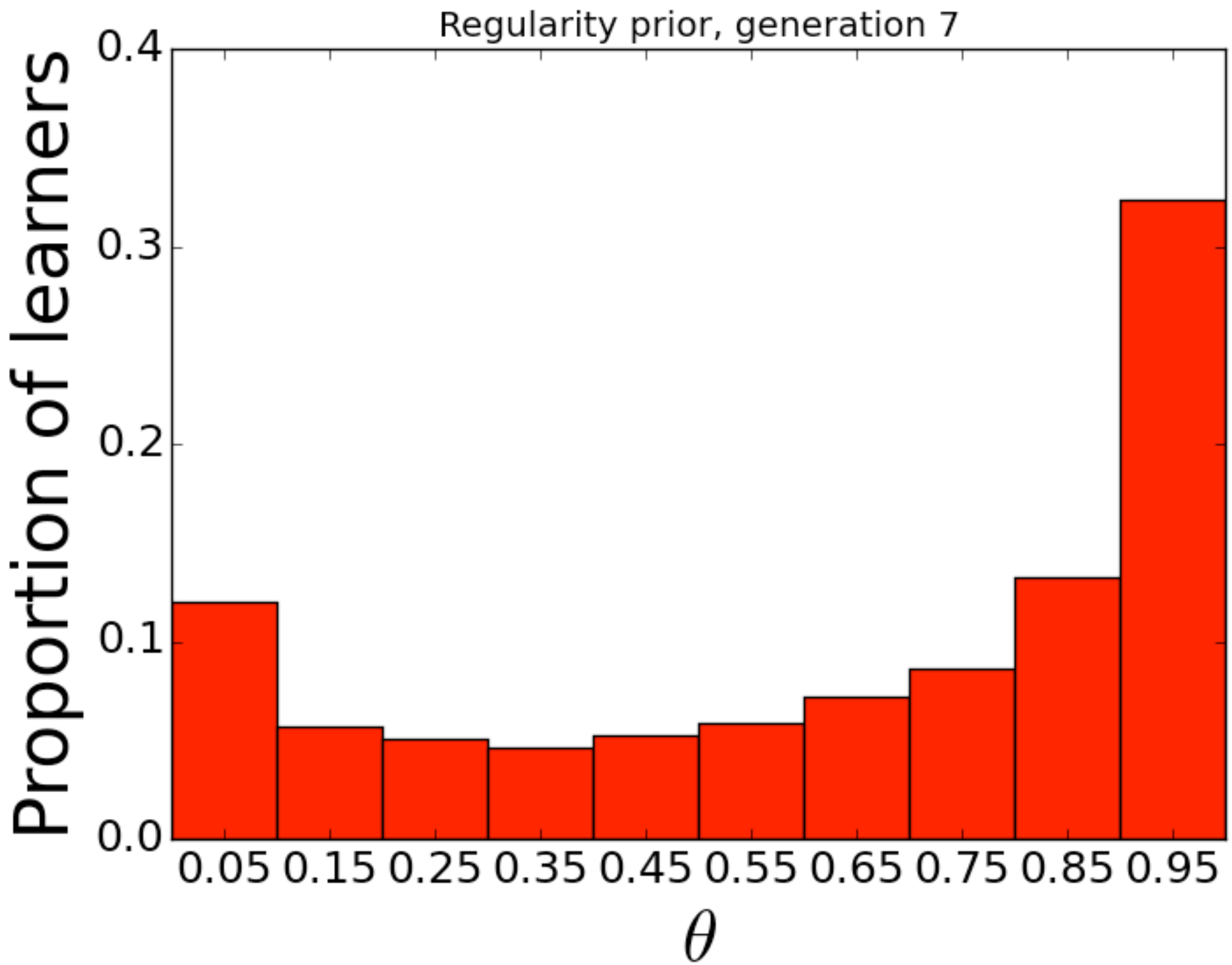

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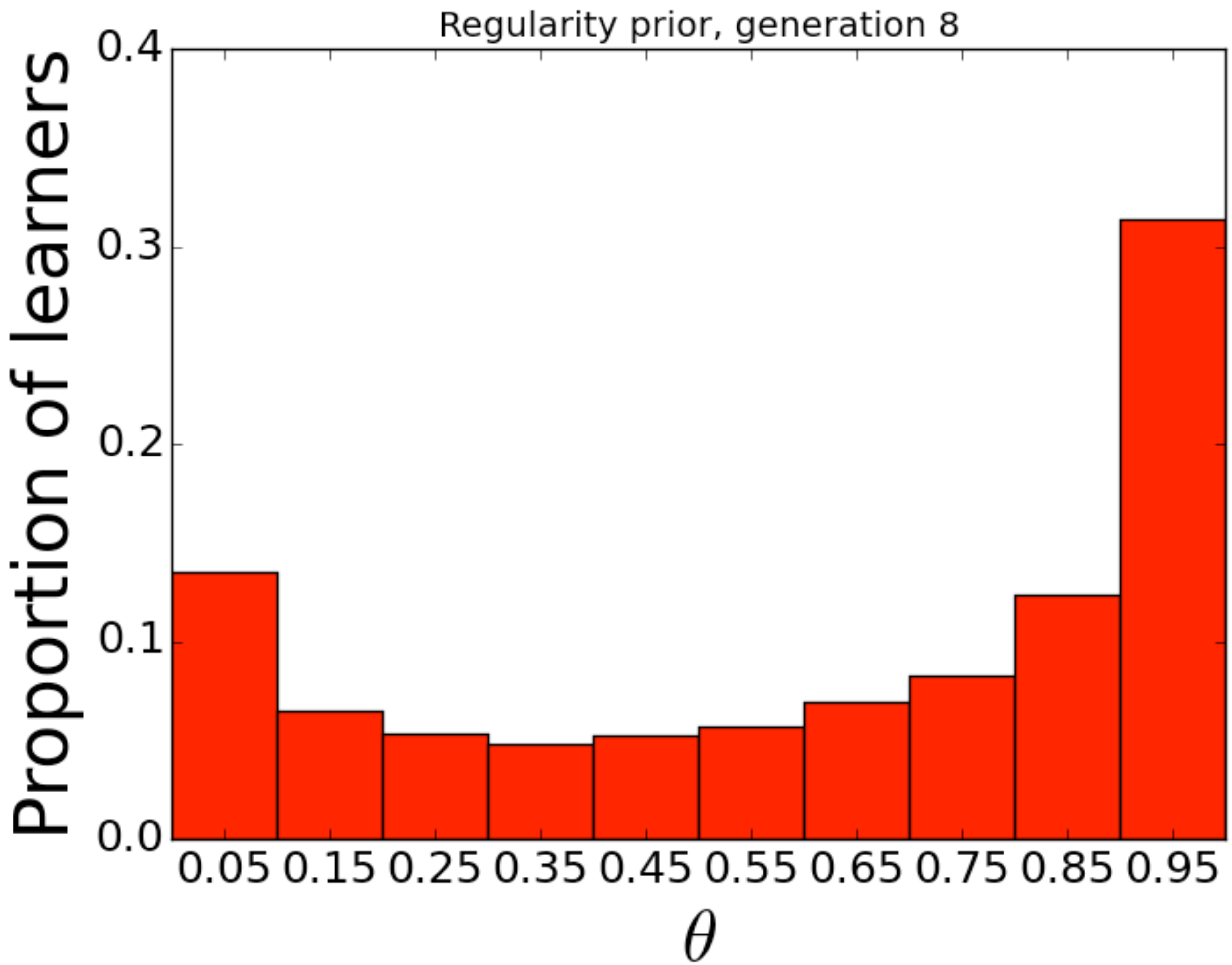

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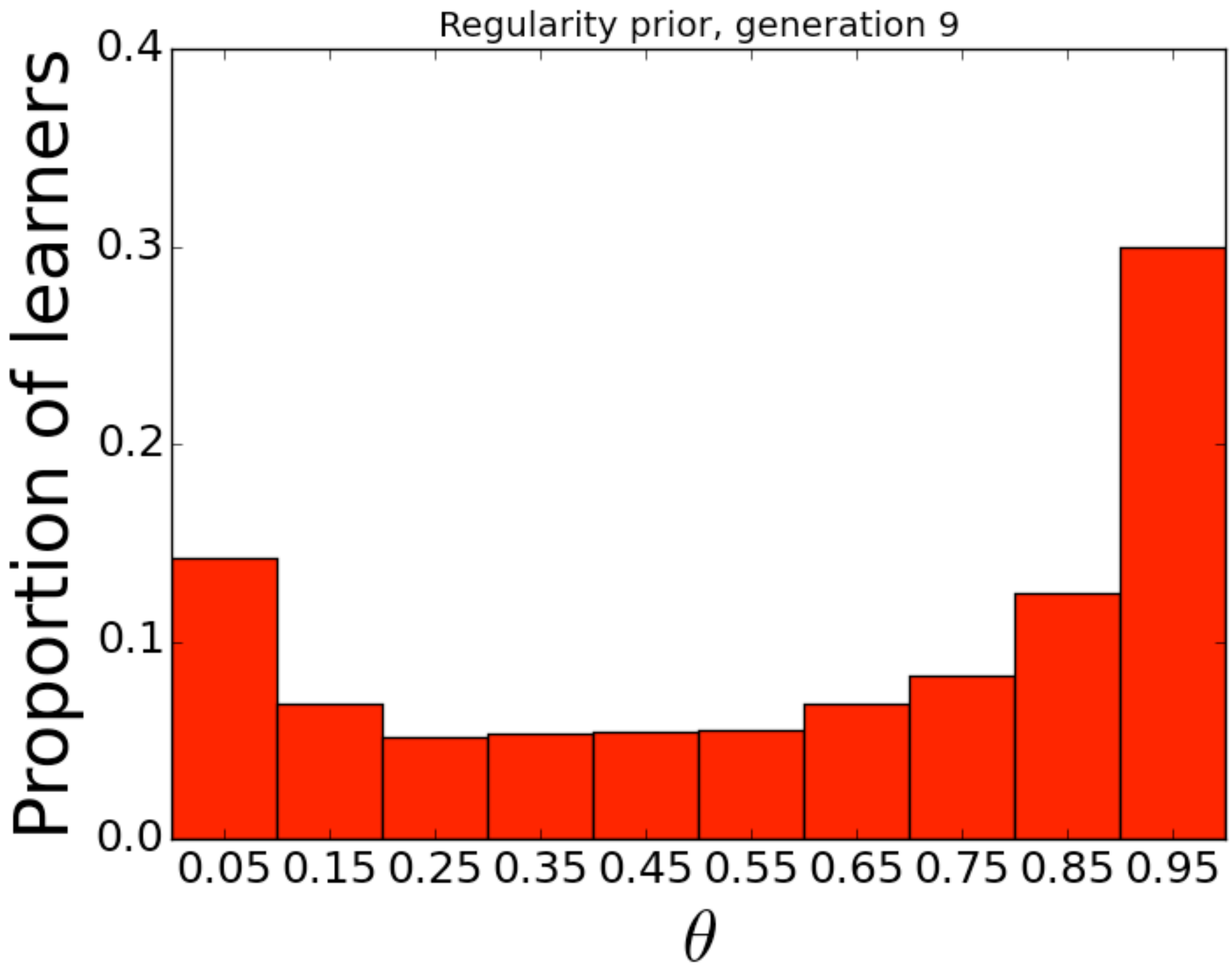

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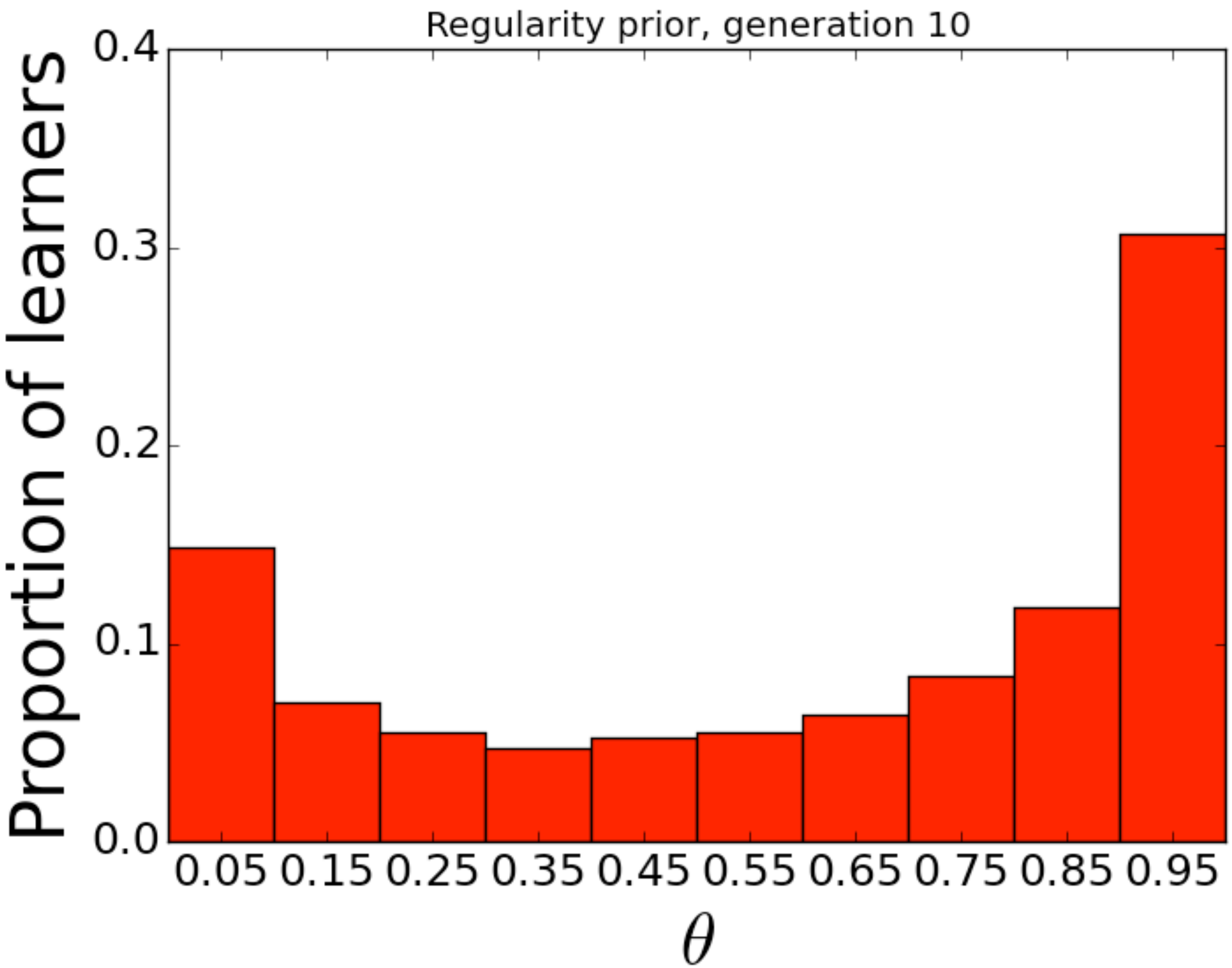

Summary and next up

$$P(h|d) \propto P(d|h)P(h)$$

- Bayesian learning: a nice simple way to model learning
- Make the bias of learners beautifully explicit
- Beta-binomial model allows us to model how learners respond to variability
- Two important insights:
  - If you study learning in individuals, data can obscure the prior
  - The prior can reveal itself over iterated learning
- Tomorrow: lab on iterated Bayesian learning
- Next week: Dr Jennifer Culbertson, more beta-binomial

References

Hudson Kam, C., & Newport, E. L. (2005). Regularizing unpredictable variation: The roles of adult and child learners in language formation and change. *Language Learning and Development, 1,* 151–195.

Reali, F., Griffiths, T. L. (2009). The evolution of frequency distributions: Relating regularization to inductive biases through iterated learning. *Cognition, 111,* 317–328.