

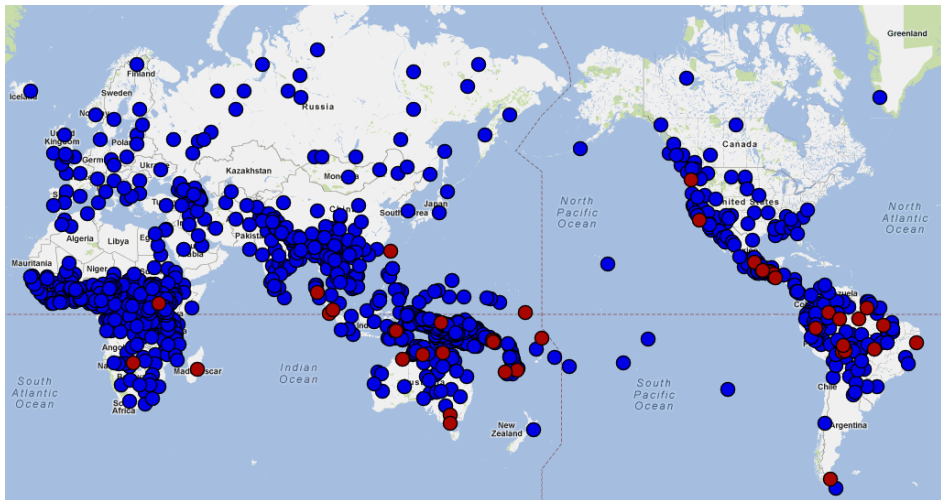
# Greenbergian Universals and Bayesian inference

Jenny Culbertson

Simulating Language, 11 March, 2015

# Greenberg's Universal 1

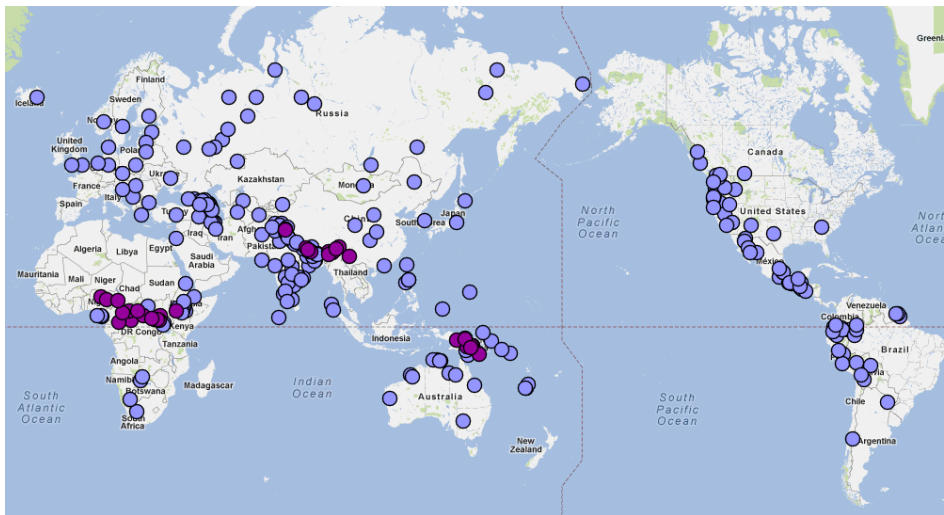
## 1. SOV, SVO, VSO (not VOS, OSV, OVS)





# Greenberg's Universal 18

## 18. If Adjective-Noun → Numeral-Noun



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  - ▶ Cognitive bias = prior bias
  - ▶ Non-uniform preference among patterns
  - ▶ (Could be innate or learned)
  - ▶ (Could be general or specialized for language)

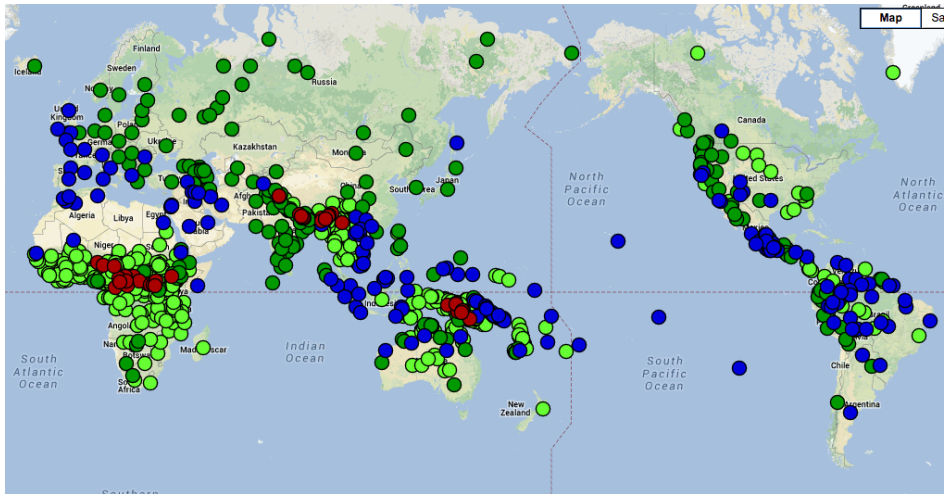
# Diversity constrained by cognitive biases?

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- ▶ Lots of confounding factors (e.g.,...?)
- ▶ But could indicate cognitive biases
  - ▶ Cognitive bias = prior bias
  - ▶ Non-uniform preference among patterns
  - ▶ (Could be innate or learned)
  - ▶ (Could be general or specialized for language)
- ▶ How to investigate? Preferences in a single generation??



# Universal 18

## 18. If Adjective-Noun → Numeral-Noun



# Universal 18

- ▶ Actually, there is more than one asymmetry here...

	N-Adj	Adj-N
Num-N	17%	27%
N-Num	52%	4%

# Universal 18

- ▶ Actually, there is more than one asymmetry here...

	N-Adj	Adj-N
Num-N	17%	27%
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- ▶ Related to another bias you've read about??

# Setting up the experiment

- ▶ The conditions

	N-Adj	Adj-N
Num-N	3	1
N-Num	2	4

- ▶ Easy or hard to learn...?

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Num-N	3	1
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- ▶ Easy or hard to learn...?
- ▶ Adding in regularization...
  - ▶ 70% dominant pattern, 30% minority pattern
  - ▶ What would regularization look like in this case?

# Formulating hypotheses

- ▶ Training = listing to Adj-N, N-Adj, Num-N, N-Num phrases
- ▶ Testing = producing phrases

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  - H2. Learners regularization variation
  - H3. Learners regularize but only orders that are easy to learn

# Formulating hypotheses

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H3. Input likelihood  $\times$  regularization prior  $\times$  order prior

# Making predictions

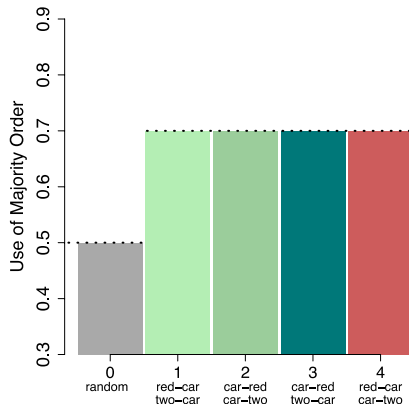
- ▶ Three predicted outcomes...



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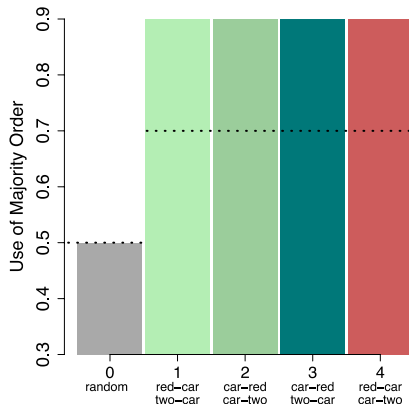
## 1. Probability matching



# Making predictions

► Three reasonable outcomes...

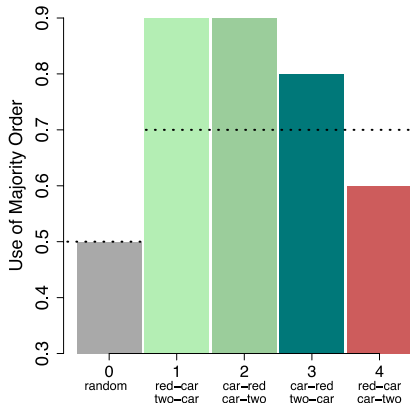
1. Probability matching
2. Across the board regularization



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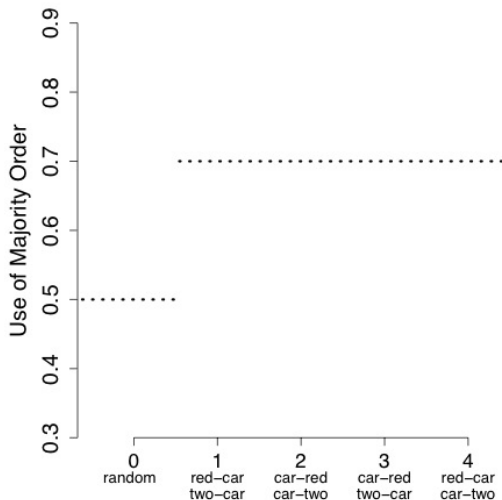
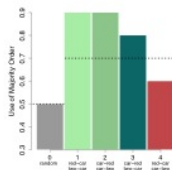
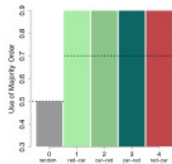
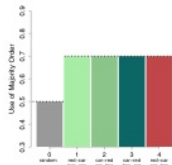
► Three reasonable outcomes...

1. Probability matching
2. Across the board regularization
3. Regularization modulated by order



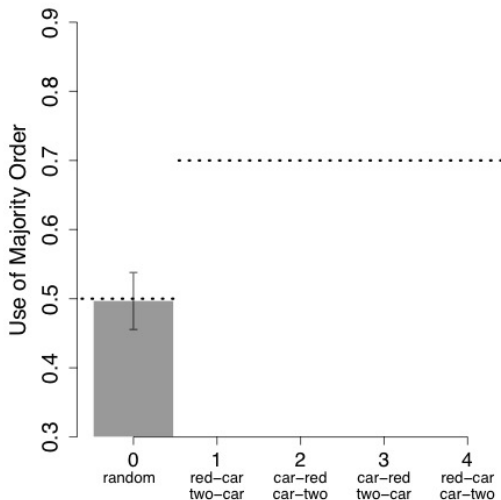
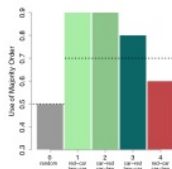
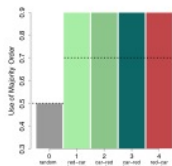
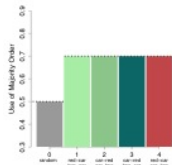
# Results

- Participants: 65 native-English-speaking undergrads



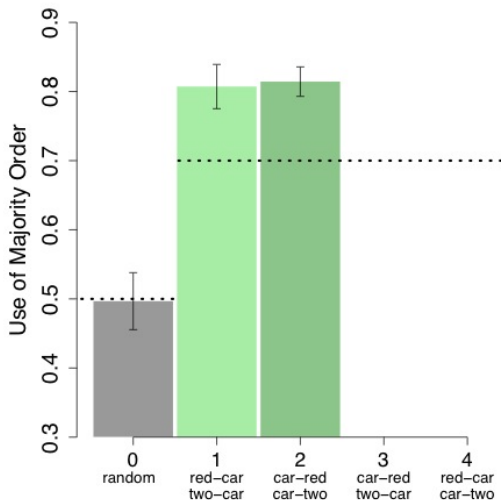
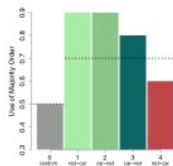
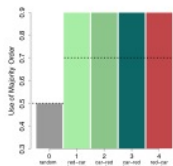
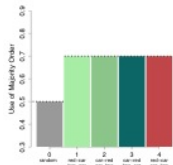
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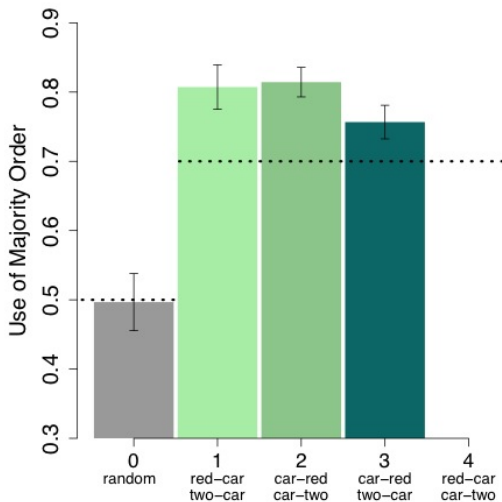
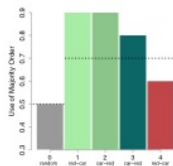
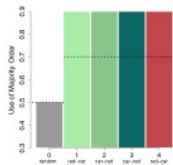
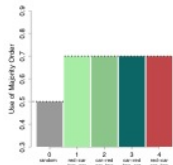
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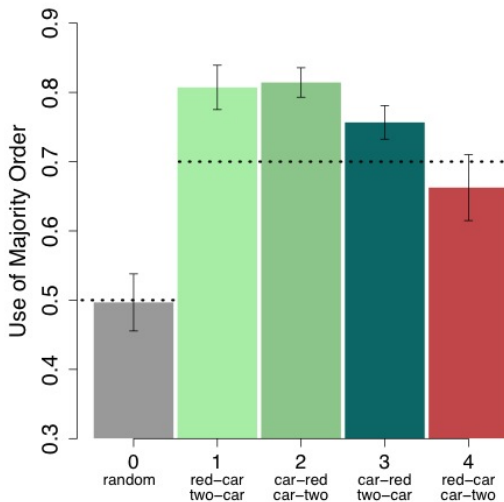
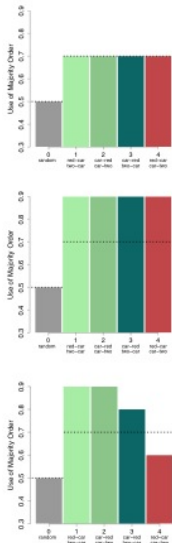
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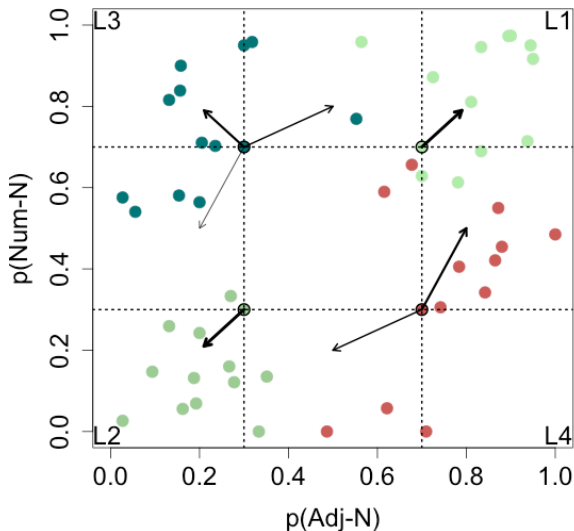
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# Individual learner outcomes



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- ▶ Likelihood
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  - ▶ How many heads out of total tosses?
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- ▶ Likelihood

$$\text{binomial}(5 \text{ heads} \mid p = 0.5, 10 \text{ tosses}) = 0.2$$

$$\text{binomial}(5 \text{ heads} \mid p = 0.9, 10 \text{ tosses}) = 0.001$$

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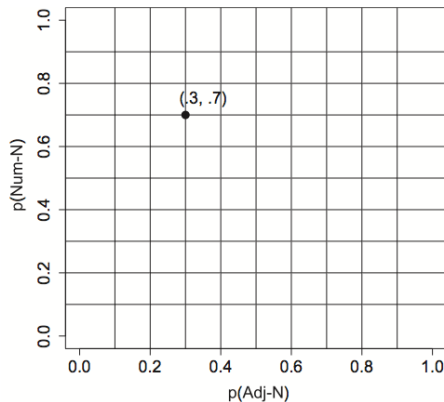
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$$\text{binomial}(28 \text{ Adj-N} \mid p = 0.3, 40 \text{ Adj}) = 0.0000018$$

# Likelihood

- ▶ Adj *and* Num ordering
  - ▶ Grid of possible probability combos
  - ▶ Each assigns likelihood to a set of counts
  - ▶ (Total likelihood just multiplies Adj and Num likelihoods)

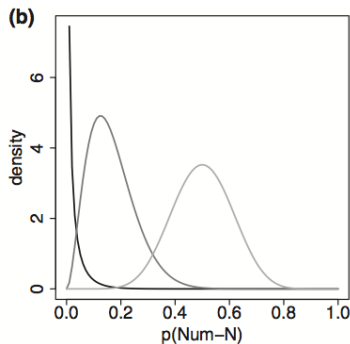
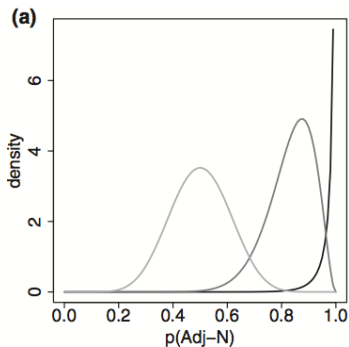


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  - H2. Input likelihood  $\times$  regularization prior  $\times$  flat order prior
  - H3. Input likelihood  $\times$  regularization prior  $\times$  biased order prior
- ▶ Likelihood
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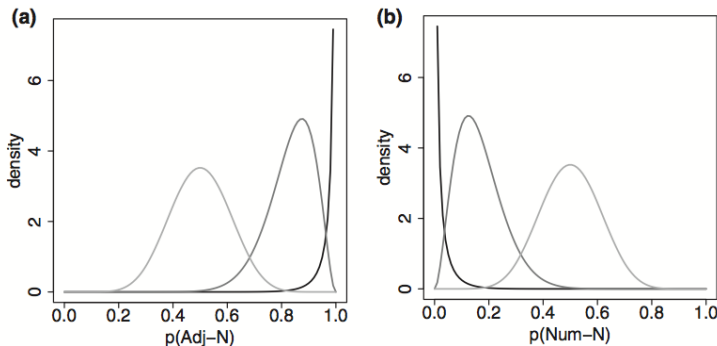
# Regularization prior

- ▶ Which points in the grid are more likely a priori?



# Regularization prior

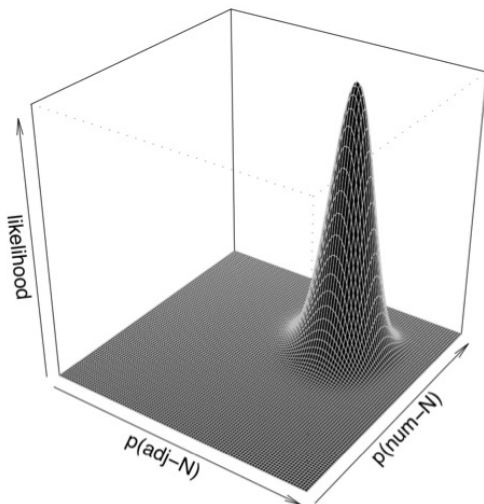
- ▶ Which points in the grid are more likely a priori?



- ▶ *Asymmetrical* beta distributions: skewed parameters  $\rightarrow$  one-way regularization

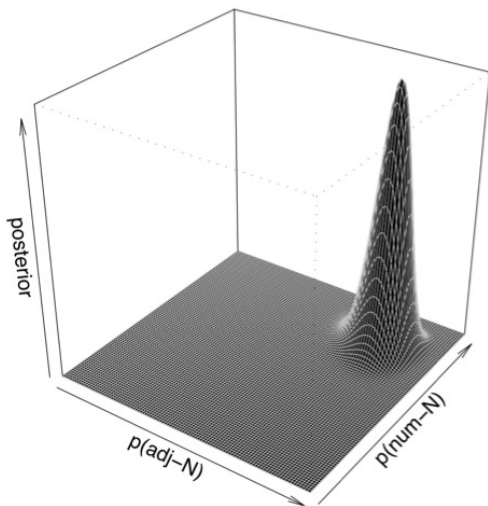
# Effect of prior on posterior

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- ▶ Which points in the grid are more likely a priori?
- ▶ Parameters of the beta:  $\alpha, \beta$
- ▶ Same as the regularization prior from Reali & Griffiths, but asymmetrical
- ▶ Conceptually: prior *counts*, e.g. of Adj-N utterances

# Formulating hypotheses

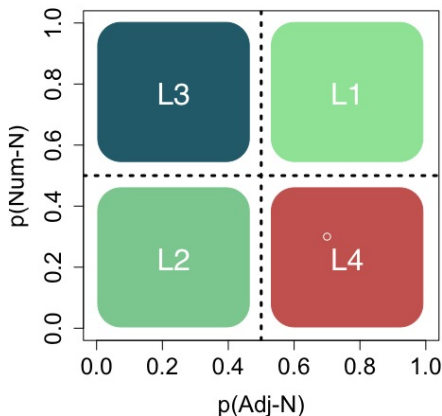
- ▶ In terms of Bayesian inference...
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  - H3. Input likelihood  $\times$  regularization prior  $\times$  biased order prior
- ▶ Likelihood
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[what would a biased one look like??]

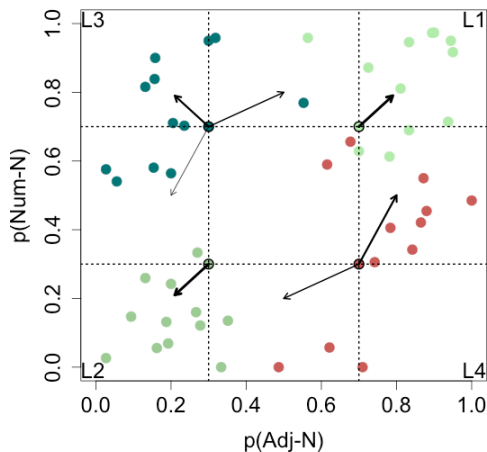
# Complete prior

- ▶ Complete prior probability of a grammar  $p(\text{Adj-N})$ ,  $p(\text{Num-N})$  is a sum over four beta combinations of:
  - ▶ prior probability of  $p(\text{Adj-N})$  given regularization bias  $\times$
  - ▶ prior probability of  $p(\text{Num-N})$  given regularization bias  $\times$
  - ▶ prior probability of particular combination of betas
- ▶ e.g., prior for  $p(\text{Adj-N})=0.8$ ,  $p(\text{Num-N})=0.2$

$$\begin{aligned} & \text{beta}(0.8|\alpha = 10, \beta = 2) \times \text{beta}(0.2|\alpha = 10, \beta = 2) \times 0.25 + \\ & \text{beta}(0.8|\alpha = 2, \beta = 10) \times \text{beta}(0.2|\alpha = 2, \beta = 10) \times 0.25 + \\ & \text{beta}(0.8|\alpha = 2, \beta = 10) \times \text{beta}(0.2|\alpha = 10, \beta = 2) \times 0.25 + \\ & \text{beta}(0.8|\alpha = 10, \beta = 2) \times \text{beta}(0.2|\alpha = 2, \beta = 10) \times 0.25 + \end{aligned}$$

# Looking for prior biases

- ▶ What parameters make the testing data most likely?



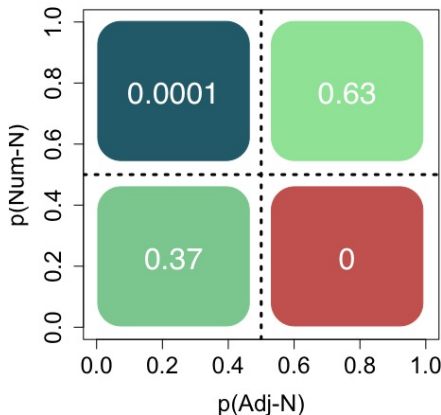


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- ▶ What parameters make the testing data most likely?
- ▶ Regularization parameters  $(\alpha, \beta)$  very skewed (16.5, 0.001)
- ▶ Prior probability of pattern types:



# Posterior (finally!)

- ▶ What kinds of  $p(\text{Adj-N})$ ,  $p(\text{Num-N})$  pairs are learners likely to acquire given set of prior parameters?

# Posterior (finally!)

- ▶ What kinds of  $p(\text{Adj-N})$ ,  $p(\text{Num-N})$  pairs are learners likely to acquire given set of prior parameters?
- ▶ Prior probability of  $p(\text{Adj-N})=\text{high}$ ,  $p(\text{Num-N})=\text{high}$  is high
- ▶ Prior probability of  $p(\text{Adj-N})=\text{low}$ ,  $p(\text{Num-N})=\text{low}$  is high
- ▶ Prior probability of  $p(\text{Adj-N})=\text{low}$ ,  $p(\text{Num-N})=\text{high}$  is pretty low
- ▶ Prior probability of  $p(\text{Adj-N})=\text{high}$ ,  $p(\text{Num-N})=\text{low}$  is zero!

	N-Adj	Adj-N
Num-N	17%	27%
N-Num	52%	4%

# For the lab...

- ▶ Calculate posterior distributions
- ▶ Recreate model predictions
- ▶ Investigate the effect of the prior parameters on predicted grammars
- ▶ Extra-credit: iterate it