# **Recursion and the infinitude claim\***

GEOFFREY K. PULLUM BARBARA C. SCHOLZ

## **1** Infinitude as a linguistic universal

In a number of recent works, linguists have portrayed the infinitude of sentences in human languages as an established linguistic universal. Lasnik (2000) asserts, in the opening chapter of a textbook based on his introductory graduate syntax lectures:

Infinity is one of the most fundamental properties of human languages, maybe the most fundamental one. People debate what the true universals of language are, but indisputably, infinity is central. (Lasnik 2000: 3)

This is not a statement about the use of idealized infinitary mathematical models in theoretical science. It is about alleged "fundamental properties of human languages" themselves.

Epstein and Hornstein (2005), a letter originally submitted for publication in *Science* but ultimately printed in *Language*, is yet bolder:

(2) This property of discrete infinity characterizes EVERY human language; none consists of a finite set of sentences. The unchanged central goal of linguistic theory over the last fifty years has been and remains to give a precise, formal characterization of this property and then to explain how humans develop (or grow) and use discretely infinite linguistic systems. (Epstein and Hornstein 2005: 4)

Here again, "discrete infinity" (which we take to mean denumerable infinity in sets of discrete elements such as symbol strings) is claimed to be a feature of "EVERY human language", as if one by one they had all been examined by scientists and checked for discrete infinitude.

Yang (2006: 103–104) takes up this theme in a popular book. After a rather confusing equation of recursion with reproduction ("Language ... has

the ability of self-reproduction, or *recursion*, to use a term from mathematics: a phrase may beget another phrase, then another, then yet another"), plus an assertion that "There is no limit on the depth of embedding", and an assertion that prepositional phrase modifiers may be added "... ad infinitum", he remarks:

(3) Recursion pops up all over language: many have argued that the property of recursive infinity is perhaps *the* defining feature of our gift for language.

(A footnote here refers the reader to Hauser et al. (2002), whose claims about recursion as the defining property of human language have been widely repeated; see Pinker and Jackendoff (2005) for further discussion.)

Remarks such as these represent infinitude as a fact about languages, which contrasts with views that were current fifty years ago. Chomsky (1957b: 15) simply remarks that a grammar projects from a finite corpus to "a set (presumably infinite) of grammatical utterances", the infinite cardinality of the projected set being treated as a side consequence of the way the theory is set up. This is precisely in line with the views of his doctoral supervisor, Zellig Harris, who stated in the same year:

Although the sample of the language out of which the grammar is derived is of course finite, the grammar which is made to generate all the sentences of that sample will be found to generate also many other sentences, and unboundedly many sentences of unbounded length. If we were to insist on a finite language, we would have to include in our grammar several highly arbitrary and numerical conditions–saying, for example, that in a given position there are not more than three occurrences of *and* between N. (Harris 1957: 208)

Harris's point is that a grammar should not include arbitrary numerical stipulations with no function other than to block coordinations from having unboundedly many coordinates. It is better, he proposes, to accept the consequence that the grammar generates unboundedly many sentences longer than any found in the corpus providing its evidential basis.

It is of course a familiar feature of science that idealizing assumptions are made, and that the idealized models have characteristics that are strictly false of the phenomena under study. Sometimes, for example, finite systems are modeled as infinite if that simplifies the mathematics. This is clearly what Harris is alluding to. And it is desirable, provided it does not result in distortion of predictions in finite domains, and it does enhance elegance in theories.

But contemporary linguists, particularly when writing for broader audiences such as beginning students, scientists in other fields, and the public at large, are treating infinitude as a property of languages themselves. This shift of view appears to stem from a kind of argument for infinitude that begins with observed facts about human language syntax and draws from them a conclusion concerning infinite cardinality.

# 2 The Standard Argument

The argument that linguists have most relied upon for support of the infinitude claim is actually a loose family of very similar arguments that we will group together and call *the Standard Argument*. Versions of it are rehearsed in, for example, Postal (1964), Bach (1964), Katz (1966), Langacker (1973), Bach (1974), Huddleston (1976), Pinker (1994), Stabler (1999), Lasnik (2000), Carnie (2002), and Hauser et al. (2002).

The Standard Argument starts with certain uncontested facts about the syntactic structure of certain classes of expressions. It draws from these the intermediate conclusion that there can be no longest expression. The infinitude claim then follows.

For concreteness, here as throughout much of the paper, we limit ourselves to English illustrations of the relevant kinds of syntactic facts. A few representative examples are given in (I).

(I) Syntactic facts

I exist is a declarative clause, and so is I know that I exist, and so is I know that I know that I exist; came in and went out is a verb phrase coordination, and so is came in, turned round, and went out, and so is came in, saw us, turned round, and went out; very nice is an adjective phrase, and so is very very nice, and so is very very very nice; and so on for many other examples and types of example.

It is not controversial that a huge collection of facts of this sort, showing grammaticality-preserving extensibility of various types of expression, could be presented for many different languages. References to (I) in what follows are intended to refer to some suitably large collection of such facts.

The intermediate conclusion that purportedly follows from facts like those in (I) is presented in (II):

(II) The No Maximal Length claim (NML)

For any English expression there is another expression that is longer. (Equivalently: No English expression has maximal length.)

Some linguists give a stronger claim we can call NML<sup>+</sup>, which entails (II): They claim not just that for any expression a longer expression always exists, but that starting from any arbitrary grammatical expression you can always *construct* a longer one that will still be grammatical, simply by adding words. NML<sup>+</sup> is never actually crucial to the argument, but we note various appearances of it below.

The ultimate conclusion from the argument is then (III):

(III) The Infinitude Claim

The collection of all grammatical English expressions is an infinite set.

Presentations of the Standard Argument utilizing (I) - (III) in various forms can be found in large numbers of introductory texts on linguistics. Langacker (1973), for example, asserts (II) as applied to English, in both its weaker and its stronger form (he seems to offer NML<sup>+</sup> as an explication of why NML must be true), and concludes (III), with an additional claim appended:

(4) There is no sentence to which we can point and say, 'Aha! This is the longest sentence of the language.' Given any sentence of English (or any other language), it is easy to find a longer sentence, no matter how long the original is ... The set of well-formed sentences of English is infinite, and the same is true of every other language. (Langacker 1973: 30)

The parenthetical remark "or any other language", claiming a universalization of (III) to all human languages, does not, of course, follow from the premises that he states (compare the similar remark by Epstein and Hornstein in (2)).

Bach (1974: 24) states that if we assent to (II) – which he gives as NML<sup>+</sup> – then we must accept (III):

(5) If we admit that, given any English sentence, we can concoct some way to add at least one word to the sentence and come up with a

longer English sentence, then we are driven to the conclusion that the set of English sentences is (countably) infinite. (1974: 24)

(The parenthesized addition "countably" does not follow from the premises supplied, but we ignore that.)

Huddleston (1976) (making reference to unbounded multiple coordination rather than subordination facts) also asserts that if we accept (II) we must accept (III):<sup>1</sup>

(6) to accept that there are no linguistic limits on the number of clauses that can be coordinated within a sentence is to accept that there are no linguistic limits on the number of different sentences in the language, *ie* that there is a (literally) infinite set of well-formed sentences. (Huddleston 1976: 7)

Stabler (1999: 321) poses the question "*Is the set of linguistic structures finite?*" as one of the issues that arises in connection with applying formal grammars to human languages, and answers it by stating that (II) seems to be true, so we can conclude (III):

(7) there seems to be no longest sentence, and consequently no maximally complex linguistic structure, and we can conclude that human languages are infinite.

A more recent discussion in Hauser, Chomsky and Fitch (2002: 1571) affirms that human languages have "a potentially infinite array of discrete expressions" because of a "capacity" that "yields discrete infinity (a property that also characterizes the natural numbers)." They proceed to the rather surprising claim that "The core property of discrete infinity is intuitively familiar to every language user" (we doubt this), and then state a coordination redundantly consisting of three different ways of expressing (III):

(8) There is no longest sentence (any candidate sentence can be trumped by, for example, embedding it in 'Mary thinks that...'), and there is no non-arbitrary upper bound to sentence length.

Other passages of a broadly similar character could be cited. We now proceed to critique the argument that they all hint at.

### **3** How the Standard Argument fails

All the linguists quoted in (4) - (8) seem to be concentrating on the step from (II) to (III), which is trivial mathematics. Under the traditional informal definition of 'infinite', where it simply means 'not finite' (a collection being finite if and only if it we can count its elements and then stop. As Dretske (1965: 100) remarks, to say that if a person continues counting forever he *will* count to infinity is coherent, but to say that at some point he *will have* counted to infinity is not. So (II) and (III) are just paraphrases. The claim is that counting the expressions of a language like English could go on forever, which is all that 'infinite' means.

It is the inference from (I) to (II) that should be critically examined. Linguists never seem to discuss that step. What licenses inferring NML from certain syntactic properties of individual English expressions?

#### 3.1 Not inductive generalization, nor mathematical induction

To begin with, we can dismiss any suggestion that the inference from (I) to (II) is an inductive generalization – an ampliative inference from a statement about certain individuals to a statement about all the members of some collection.

An example of inductive generalization on English expressions – and a justifiable one – would be to reason from English adjective phrases like *very nice*, *very very nice*, *very very nice*, and so on, to the generalization that repeatable adverb modifiers in adjective phrases always precede the head.

But inferring that the collection of all possible English adjective phrases has no longest member is an entirely different matter. The conclusion is not about the properties of adjective phrases at all. It concerns a property of a different kind of object: It attributes a *size* to the set of all adjective phrases of a certain form, which is very different from making a generalization about their form.

A different possibility would be that (II) can be concluded from (I) by means of some kind of mathematical argument, rather than an inductive generalization from linguistic data. Pinker (1994: 86) explicitly suggests as much:

By the same logic that shows that there are an infinite number of integers–if you ever think you have the largest integer, just add 1

to it and you will have another-there must be an infinite number of sentences.

This reference to a "logic that shows that there are an infinite number of integers" is apparently an allusion to reasoning by mathematical induction.

Arguments by mathematical induction use recursion to show that some property holds of all of the infinitely many positive integers. There are two components: A base case, in which some initial integer such as 0 or 1 is established as having a certain property P, and an inductive step in which it is established that if any number n has P then n + 1 must also have P. The conclusion that every positive integer has P then follows.

However, it follows only given certain substantive arithmetical assumptions. Specifically, we need two of Peano's axioms: The one that says every integer has a successor (so there is an integer n + 1 for every n), and the one that says the successor function is injective (so distinct numbers cannot share a successor).<sup>2</sup>

Pinker's suggestion seems to be that a mathematical induction on the set of lengths of English expressions will show that English is an infinite set. This is true, provided we assume that the analogs of the necessary Peano axioms hold on the set of English expressions. That is, we must *assume* both that every English expression length has a successor, and that no two English expression lengths share a successor. But to assume this is to assume the NML claim (II). (There cannot be a longest expression, because the length of any such expression would have to have a successor that was not the successor of any other expression length, which is impossible.)

Thus we get from (I) to (II) only by assuming (II). The argument makes no use of any facts about the structure of English expressions, and simply assumes what it was supposed to show.

#### 3.2 Arguing from generative grammars

A third alternative for arguing from (I) to (II) probably comes closest to reconstructing what some linguists may have had in mind. If facts like those in (I) inevitably demand representation in terms of generative rule systems with recursion, infinitude might be taken to follow from that. The enormous influence of generative grammatical frameworks over the past fifty years may have led some linguists to think that a generative grammar *must* be posited to describe data sets like the ones illustrated in (I). If in the face of such sets of facts there was simply no alternative to assuming a generative grammar description with recursion in the rule system, then a linguistically competent human being would have to mentally represent "a recursive procedure that generates an infinity of expressions" (2002: 86–87), and thus (II) would have to be, in a sense, true.

There are two flaws in this argument. The less important one is perhaps worth noting in passing nonetheless. It is that assuming a generative framework, even with non-trivially recursive rules, does not entail NML, and thus does not guarantee infinitude. A generative grammar can make recursive use of non-useless symbols and yet not generate an infinite stringset. Consider the following simple context-sensitive grammar (adapted from one suggested by András Kornai):

(9) Nonterminals: S, NP, VP  
Start symbol: S  
Terminals: They, came, running  
Rules: 
$$S \rightarrow NP \ VP$$
  
 $VP \rightarrow VP \ VP$   
 $NP \rightarrow They$   
 $VP \rightarrow came / They \_$   
 $VP \rightarrow running / They came \_$ 

The rule "VP  $\rightarrow$  VP VP" is non-trivially recursive – it generates the infinite set of all binary VP-labelled trees. No non-terminals are unproductive (incapable of deriving terminal strings) or unreachable (incapable of figuring in a completed derivation from S). And no rules are useless – in fact all rules participate in all derivations that terminate. Yet only two strings are generated: *They came*, and *They came running*. The structures are shown in (10).



No derivation that uses the crucial VP rule more than once can terminate. Thus recursion does not guarantee infinitude.

One might dismiss this as an unimportant anomaly, and say that a proper theory of syntactic structure should simply rule out such failures of infinitude by stipulation. But interestingly, for a wide range of generative grammars, including context-sensitive grammars and most varieties of transformational grammar, questions of the type 'Does grammar G generate an infinite set of strings?' are undecidable, in the sense that no general algorithm can determine whether the goal of "a recursive procedure that generates an infinity of expressions" has been achieved. One could stipulate in linguistic theory that the permissible grammars are (say) all and only those context-sensitive grammars that generate infinite sets, but the theory would have the strange property that whether a given grammar conformed to it would be a computationally undecidable question.<sup>3</sup>

One important point brought out by example grammars like (9) is that you can have a syntax that generates an infinitude of structures without thereby having an infinitude of generated expressions. Everything depends on the lexicon. In (9) only the lexical items *They, came,* and *running* are allowed, and they are in effect subcategorized to ensure that *came* has to follow *They* and *running* has to follow *came.* Because of this, almost none of the rich variety of subtrees rooted in VP can contribute to the generation of strings. Similarly, the syntax of a human language could allow clausal complementation, but if the lexicon happened to contain no relevant lexical items (verbs of propositional attitude and the like), this permissiveness would be to no avail.

However, there is a much more important flaw in the argument via generative grammars. It stems from the fact that generative grammars are *not* the only way of representing data such as that given in (I). There are at least three alternatives – non-generative ways of formulating grammars that are mathematically explicit, in the sense that they distinguish unequivocally between grammatical and ungrammatical expressions, and model all of the structural properties required for well-formedness.

First, we could model grammars as transducers, i.e., formal systems that map between one representation and another. It is very common to find theoretical linguists speaking of grammars as mapping between sounds and meanings. They rarely seem to mean it, because they generally endorse some variety of what Seuren (2004) calls *random generation* grammars, and Seuren is quite right that these cannot be regarded as mapping meaning to sound. For example, as Manaster Ramer (1993) has pointed out, Chomsky's remark that a human being's internalized grammar "assigns a status to every relevant physical event, say, every sound wave" (Chomsky 1986: 26) is false of the generative grammars he recognizes in the rest of that work: Grammars of the sort he discusses assign a status only to strings that they generate. They do not take inputs; they merely generate a certain set of abstract objects, and they cannot assign linguistic properties to any object not in that set. However, if grammars were modeled as transducers, grammars *could* be mappings between representations (e.g., sounds and meanings), without regard to how many expressions there might be. Such grammars would make no commitment regarding either infinitude or finitude.

A second possibility is suggested by an idea for formalizing the transformational theory of Zellig Harris. Given what Harris says in his various papers, he might be thought of as tacitly suggesting that grammars could be modeled in terms of category theory. There is a collection of objects (the utterances of the language, idealized in Harris 1968 as strings paired with acceptability scores), whose exact boundaries are not clear and do not really matter (see Harris 1968: 10-12 for a suggestion that the collection of all utterances is "not well-defined and is not even a proper part of the set of word sequences"); and there is a set of morphisms defined on it, the transformations, which appear to meet the defining category-theoretic conditions of being associative and composable, and including an identity morphism for each object. In category theory the morphisms defined on a class can be studied without any commitment to the cardinality of the class. A category is characterized by the morphisms in its inventory, not by the objects in the underlying collection. This seems very much in the spirit of Harris's view of language, at least in Harris (1968), where a transformation is "a pairing of sets ... preserving sentencehood" (p. 60).

Perhaps the best-developed kind of grammar that is neutral with respect to infinitude, however, is a third type: The purely constraint-based or modeltheoretic approach that has flourished as a growing minority viewpoint in formal syntax over the past thirty years, initially given explicit formulation by Johnson and Postal (1980) but later taken up in various other frameworks — for example, LFG as presented in Kaplan (1995) and as reformalized by Blackburn and Gardent (1995); GPSG as reformalized by Rogers (1997); and HPSG as discussed in Pollard (1999) and Ginzburg and Sag (2000).

The idea of constraints is familiar enough within generative linguistics. The statements of the binding theory in GB (Chomsky, 1981), for example, entail nothing about expression length or set size. (To say that every anaphor is bound in its governing category is to say something that could be true regardless of how many expressions containing anaphors might exist.) Chomsky (1981) used such constraints only as filters on the output of an underlying generative grammar with an X-bar phrase structure base component and a movement transformation. But in a fully model-theoretic framework, a grammar consists of constraints on syntactic structures and *nothing more* – there is no generative component at all.

Grammars of this sort are entirely independent of the numerosity of expressions (though conditions on the class of intended models can be stipulated at a meta-level). For example, suppose the grammar of English includes statements requiring (i) that adverb modifiers in adjective phrases precede the head adjective; (ii) that an internal complement of *know* must be a finite clause or NP or PP headed by *of* or *about*; (iii) that all content-clause complements follow the lexical heads of their immediately containing phrases; and (iv) that the subject of a clause precedes the predicate. Such conditions can adequately represent facts like those in (I). But they are compatible with *any* answer to the question of how many repetitions of a modifier an adjective can have, or how deep embedding of content clauses can go, or how many sentences there are. The constraints are satisfied by expressions with the relevant structure whether there are infinitely many of them, or a huge finite number, or only a few.

#### 3.3 Interim summary

We have made four points so far. First, the inference from (I) to (II) is not a cogent inductive (ampliative) generalization. Second, it can be represented as a deductive argument (a mathematical induction on the integers) only at the cost of making it completely circular. Third, requiring that human languages be modeled by generative grammars with recursive rule systems does not in fact guarantee infinitude. And fourth, it is not necessary to employ generative grammars in order to model the data of (I) – there are at least three other kinds of fully explicit grammars that are independent of how many expressions there are.

Of course, linguists certainly have the right to simply assume (III) – or equivalently (II) – as an axiom. But it is quite hard to see why they should want to. This would be an unmotivated axiom with no applications. It neither entails generative grammars with recursion nor is entailed thereby. With no consequences for linguistic structure, and no consequences for human knowl-

edge of linguistic structure, it would appear to be an unnecessary excrescence in syntactic theory (and incidentally, one that is not first-order expressible).

## **4** The stubborn seductiveness of infinitude

If the Standard Argument for infinitude fails so clearly, and the property itself has no discernible applications in linguistics, the question arises of why the conclusion of the argument has been so seductive to so many linguists. We briefly consider four factors that seem to have contributed to linguists' eagerness to believe in language infinitude despite its singular inertness in actual linguistic practice.

#### 4.1 The notion that languages are collections

There can be no doubt that one factor tempting linguists to accept infinitude is the ubiquitous presupposition that a language is appropriately given a theoretical reconstruction as a collection of expressions. This is not an ordinary common-sense idea: Speakers never seem to think of their language as the collection of all those word sequences that are grammatically well-formed. The idea of taking a language as a set of properly structured formulae stems from mathematical logic. Its appearance in generative grammar and theoretical computer science comes from that source. It is alien to the other disciplines that study language (anthropology, philology, sociolinguistics, and so on).<sup>4</sup>

The source of the idea that a language is a collection of word sequences lies in early generative grammar, with its emphasis on processes of derivation and its origins in the theory of recursively enumerable sets of symbol sequences. It placed an indelible stamp on the way linguists think about languages. It has even survived direct rejection by Chomsky (1986: 20ff), where the term 'E-language' is introduced to cover any and all views about language that are "external" to the mind – not concerned with 'I-language' (languages construed as "internal", "individual", and "intensional"). 'E-language' covers all sorts of traditional views such as that a language is a socially shared system of conventions, but also the mathematical conception of a language as an infinite set of finite strings.

Chomsky's dismissal of the notion of infinite sets of sentences as irrelevant to modern linguistics leaves no place at all for claims about the infinitude of languages. Chomsky dismisses the study of sets of expressions (e.g., weak generative capacity studies) for "its apparent uselessness for the theory of language." But questions of cardinality only sensibly apply to the conception of language that Chomsky rejects as useless. It is hard to see any application for mathematical results on infinite sets in the study of a biological object like a brain component.

Linguists and philosophers who follow Chomsky's terminology assert that they study 'I-language' rather than 'E-language'. But the view of languages as collections has persisted anyway, even among those who purport to believe it atavistic. If a language is a set of expressions, it has to be either finite or infinite; and if taking it to be finite is out of the question, then (if it is finitely describable at all) it can only be a computably enumerable infinite set. But these are conditional claims. The infinitude claim depends crucially on a prior decision to stipulate that a language has to be a set.

Chomsky comes quite close to expressing the alternative view that we urge when he includes "intensional" in his characterization of 'I-language'. The goal of a grammar is not to reconstruct a language extensionally, as a collection containing all and only the well-formed expressions that happen to exist; rather, a grammar is about structure, and linguistic structure should be described intensionally, in terms of constraints representing the form that expressions share. That does not require that we regard any particular set, finite or infinite, as the one true set that corresponds to a particular grammar or stands as the unique theoretical representation of a particular human language.

Linguists' continued attraction toward the idea that languages are infinite is thus at least in part an unjustified hangover from the extensional view adopted by mathematical logicians in the work that laid the mathematical foundations of generative grammar (on which see Scholz and Pullum 2007).

#### 4.2 The phenomenon of linguistic creativity

A second factor that encourages linguists to believe that human languages are infinite sets stems from a presumed connection between linguistic creativity and the infinite cardinality of languages. Note, for example, this statement by Chomsky (1980: 221–222):

... the rules of the grammar must iterate in some manner to generate an infinite number of sentences, each with its specific

sound, structure, and meaning. We make use of this "recursive" property of grammar constantly in everyday life. We construct new sentences freely and use them on appropriate occasions...

He is suggesting that because we construct new sentences, we must be using recursion, so the grammar must generate infinitely many sentences. Note also the remark of Lasnik (2000: 3) that "The ability to produce and understand new sentences is intuitively related to the notion of infinity."

No one will deny that human beings have a marvelous, highly flexible array of linguistic abilities. These abilities are not just a matter of being able to respond verbally to novel circumstances, but of being capable of expressing novel propositions, and of re-expressing familiar propositions in new ways. But infinitude of the set of all grammatical expressions is neither necessary nor sufficient to describe or explain linguistic creativity.

To see that infinitude is not necessary (and here we are endorsing a point made rather differently in the philosophical literature by Gareth Evans 1981), it is enough to notice that creating a verse in the very tightly limited Japanese *haiku* form (which can be done in any language) involves creation within a strictly finite domain, but is highly creative nonetheless, seemingly (but not actually) to an unbounded degree. Over a fixed vocabulary, there are only finitely many haiku verses. Obviously, the precise cardinality does not matter: The range is vast. A haiku in Japanese is composed of 17 phonological units called morae, and Japanese has roughly 100 morae (Bill Poser, personal communication), so any haiku that is composed is being picked from a set of phonologically possible ones that is vastly smaller than  $100^{17} = 10^{34}$  (given that phonotactic, morphological, lexical, syntactic, semantic, pragmatic, and esthetic considerations will rule out most of the pronounceable mora sequences).

This set is large enough that competitions for haiku composition could proceed continuously throughout the entire future history of the human race, and much longer, without a single repetition coming up accidentally. That is part of what is crucial for making haiku construction creative, and gives the poet the experience of creativity. The range of allowable possibilities (under the constraints on form) should be vast; but it does not need to be infinite.

Indeed, language infinitude is not only unnecessary but is also insufficient for linguistic creativity. For mere iterable extension of expression length hardly seems to deserve to be called creative. Take the only recursive phrase structure rule in *Syntactic Structures* (Chomsky 1957b, where embedding of subordinate clauses was accomplished differently, by generalized transformations), quoted above: The rule "Adj  $\rightarrow$  very Adj". If that rule is included in a generative grammar that generates at least one string where some lexical item appears as an expansion of Adj, then the set of generated strings is infinite. Over the four-word vocabulary {*John, is, nice, very*}, for example, we get an infinite number of sentences like *John is very, very, very, very, ..., very nice.* Infinitude, yes, under the generative idealization. But creativity? Surely not.

Repetitiveness of this sort is widely found in aspects of nature where we would not dream of attributing creativity: A dog barking repeatedly into the night; a male cricket in late summer desperately repeating its stridulational mating call over and over again; even a trickle of water oozing through a cave roof and dripping off a stalactite has the same character. All of them could be described by means of formal systems involving recursion, but they provide no insight into or explication of the kind of phenomena in which human linguistic creativity is manifested.

#### 4.3 The critique of associationist psychology

We conjecture that a third factor may have had some influence on linguists' enthusiasm for infinitude. A prominent feature of the interdisciplinary literature that arose out of the early generative grammar community was a broad attack on such movements as associationism and Skinnerian behaviorism in 20th-century psychology. A key charge against such views was that they could never account for human linguistic abilities because they conceived of them in terms of finite behavioral repertoires and could never explain how humans could learn, use, or understand an infinite language. Asserting the infinitude claim might thus have had a rhetorical purpose in the 1950s. The point would have been to stress that the dominant frameworks for psychological research at that time stood no chance of being able to model human linguistic capacities.

We say only that we "surmise" that such a rhetorical strategy "may have had some influence", because in fact instances of the strategy being explicitly pursued are thin on the ground. Chomsky (1957b: 26–27n) does point out that the examples of axiomatic grammars provided by Harwood (1955) "could not generate an infinite language with a finite grammar", but not in connection with any psychological point. In his critique (Chomsky, 1957a) of the psycholinguistic proposals of Hockett (1955), who gave (pp. 7–8) an illustrative stochastic generative grammar of 'select one word from each column' form, inherently limited to a finite number of outputs, Chomsky does not mention infinitude, but concentrates on the stochastic aspects of the proposal. And he ignores infinitude when citing the remarks of Lashley (1951) about how "any theory of grammatical structure which ascribes it to direct associative linkage of the words of the sentence, overlooks the essential structure of speech" (Chomsky 1958: 433).

It is just as well that infinitude was rarely used as a stick with which to beat associationism, because such a strategy would be entirely misguided. To see why, note that associationist psychology can be mathematically modeled by grammars generating sets of strings of behavioral units (represented by symbols), and the relevant grammars are the ones known as *strictly local* (SL) grammars (see Rogers and Pullum 2007, and Rogers and Hauser, this volume, for a review of the mathematics). SL grammars are nothing more than finite sets of *n*-tuples of (crucially) terminal symbols. If n = 2 a grammar is a set of bigrams, as used in various contemporary connectionist systems and speech recognition programs, and we get the SL<sub>2</sub> languages. The 'Wickelphones' of Rumelhart and McClelland 1986 are trigrams, and characterize the SL<sub>3</sub> languages. The SL class is the union of the SL<sub>n</sub> sets of grammars for all  $n \ge 2$ .

Bever, Fodor and Garrett (1968: 563) claim to provide a formal refutation of associationist psychology by pointing out (correctly) that SL grammars are not adequate for the description of certain syntactic phenomena in English. They stress the issue of whether non-terminal symbols are allowed in grammars, and remark that associationism is limited to "rules defined over the 'terminal' vocabulary of a theory, i.e., over the vocabulary in which behavior is described", each rule specifying "an n-tuple of elements between which an association can hold", given in a vocabulary involving "description of the actual behavior". SL grammars have precisely this property of stating the whole of the grammar in the terminal vocabulary. They cite a remark by Lashley (1951) which in effect observes that  $SL_2$  cannot even provide a basis for modeling the behavior seen in typing errors like typing 'Lalshey' for 'Lashley'.

However, no matter what defects SL grammars might have, an inability to represent infinite languages is not one of them. Bever et al. tacitly acknowledge this, since they nowhere mention infinitude as a problem.

We make this point because Lasnik (2000: 12) claims that the finite-state conception of grammar "is the simplest one that can capture infinity", and this

is a mistake. The infinite set we just alluded to above, containing all strings of the form 'John is (very)\* nice', is easy to represent with an  $SL_2$  grammar. (Using ' $\rtimes$ ' to mark a left sentence boundary and ' $\ltimes$ ' to mark a right boundary, the bigrams needed are: ' $\rtimes$  John', 'John is', 'is nice', 'is very', 'very very', 'very nice', and 'nice  $\ltimes$ '.)

Insofar as associationist psychology and connectionist models of cognition are theoretically reconstructible as (possibly stochasticized) varieties of  $SL_2$  grammars (or  $SL_k$  grammars, for any fixed k), they are entirely untouched by the infinitude claim. The putative infinitude of languages has no more consequences for these research programs than it does for other theories of grammar, or for linguistic creativity.

It should be kept in mind, in any case, that if creative sentence production is the topic of interest, the use of subordination considered to be typical (and educationally prized) in educated written Standard English is actually quite rare in real-life colloquial language use. Pawley and Syder (2000) argues that clausal subordination hardly occurs at all in spontaneous English speech. Quite a bit of what might be taken for on-the-fly hypotaxis is in fact fill-inthe-blanks use of semi-customizable schemata containing subordination (*It depends what you mean by* \_\_\_\_; *I can't believe* \_\_\_\_; etc.). Active spontaneous management of clause subordination in colloquial speech may be rather rare in any language. It should not be too surprising if in some preliterate cultures the resources for it are entirely absent from the grammatical system.

## 5 Finite human languages?

The quotations from Langacker (1973) and Epstein and Hornstein (2005) in (4) and (2) baldly assert that infinitude holds for every human language. And Lasnik (2000), Hauser et al. (2002), and Yang (2006) hold that infinitude, which they take to be a direct consequence of recursion in grammars, is a central and fundamental aspect of human language. We have argued that such claims are not well supported, either directly or indirectly.

As a side consequence of the stress on language infinitude and recursion, a controversy has recently emerged about the Amazonian language Pirahã, stimulated primarily by the assertion in Everett (2005) concerning its lack of both recursive hypotaxis and syndetic coordination. But Everett's claims should not surprise anyone who has a thorough acquaintance with syntactic typology. Similar properties have been attributed to many other languages, in diverse families.

Collinder (1960: 250-251, in a chapter contributed by Paavo Ravila) states the following about Proto-Uralic (PU) and many of its modern descendants:

In PU there was no hypotaxis in the strict sense of the word. The sentences were connected paratactically, and in the absence of conjunctions, the content determined the mutual relations of the sentences. In PU, as in the Uralic languages spoken today, the subordinate clauses of the Indo-European languages had as counterparts various constructions with verb nouns ...

But this differs very little from what has often been said about Australian Aboriginal languages. They show remarkably little evidence of subordination, with hardly any real indication of recursion, no finite clause complements at all, and no syndetic clause coordination. One should not be misled by the fact that structural representations showing clauses embedded within clauses in Dyirbal appear in Dixon (1972: 147-220). These are purely theoretical posits, suggested by the kinds of derivations that were assumed for English sentences at the time he was writing. Full clausal hypotaxis is never encountered in the derived structures, and the examples given seem highly paratactic, with English glosses like "Man told woman: Light fire: Find girls" (1972: 165). Moreover, Dyirbal has no syndetic coordination: there are no equivalents of words like and, or, and but. Things are similar with Wargamay: although Dixon (1981: 70) has a section headed 'complement clauses'. the examples given are not complements at all; all are clearly non-finite adjunct phrases of result or purpose. There are no finite subordinate clauses, and Dixon offers no indications of any kind of recursion.

Derbyshire (1979) describes the Amazonian language Hixkaryána (in the Cariban family, unrelated to Pirahã), and similar syntactic characteristics emerge. Hixkaryána has no finite complement clauses, hence no indirect speech constructions or verbs of propositional attitude. According to Derbyshire, "Subordination is restricted to nonfinite verbal forms, specifically derived nominals" or "pseudo-nominals that function as adverbials", and "There is no special form for indirect statements such as 'he said that he is going'..." (p. 21). (There is a verb meaning 'say' that allows for directly quoted speech, but that does not involve subordination.) Hixkaryána has nominalization (of an apparently non-recursive kind), but no clausal subordination. Derbyshire

also notes (1979: 45) the absence of any "formal means ... for expressing coordination at either the sentence or the phrase level, i.e. no simple equivalents of 'and', 'but' and 'or'."

Givon (1979: 298) discusses the topic in general terms and relates it to language evolution, both diachronic and phylogenetic. He claims that "di-achronically, in extant human language, subordination *always* develops out of earlier loose coordination", the evidence suggesting that it "must have also been a phylogenetic process, correlated with the increase in both cognitive capacity and sociocultural complexity", and he observes:

there are some languages extant to this day – all in preindustrial, illiterate societies with relatively small, homogeneous social units – where one could demonstrate that subordination does not really exist, and that the complexity of discourse–narrative is still achieved via "chaining" or coordination, albeit with an evolved discourse-function morphology...

Other works, more recent but still antedating Everett, could be cited. For example, Deutscher (2000, summarized in Sampson 2009) claims that when Akkadian was first written it did not have finite complement clauses, though later in its history it developed them.

We are not attempting an exhaustive survey of such references in the literature. We merely note that various works have noted the absence of iterable embedding in various human languages, and for some of those it has also been claimed that they lack syndetic sentence coordination (that is, they do not have sentence coordination that is explicitly marked with a coordinator word, as opposed to mere juxtaposition of sentences in discourse).

Languages with neither iterable embedding nor unbounded coordination would in principle have just a finite (though very large) number of distinct expressions, for any given finite fixing of the lexicon – (though saying that there are only finitely many sentence presupposes that we have some way of distinguishing sentences from sentence sequences; this is by no means a trivial codicil, but let us assume it).

The suggestion that there might be finite human languages, then, did not suddenly emerge with Everett's work in 2005. It is implicit in plenty of earlier linguistic literature.

No one should think that finiteness in this sense would imply some sort of inferiority for the speakers of the language, or their linguistic or cognitive abilities, or their linguistic creativity. As we argued earlier, there is no necessary link between infinitude and linguistic creativity, since for creativity the number of possible expressions needs only to be very large – it does not need to be infinite.

Nor should anyone think that finiteness of the set of sentences would imperil either the claimed semantic universal that all human languages have what Katz (1978) called 'effability' (the property of having resources for expression of all propositions) or the closely related claim of universal intertranslatability (that any proposition expressible in one language is also expressible in all others). These universal claims may be false — we take no stand either way on that — but they would not be falsified by the mere existence of some human language with only finitely many sentences.

A language that *can* express multiple thoughts in a single sentence is not thereby *required* to do so. Complex sentences involving can be re-expressed in sequences of syntactically simpler sentences. For example, *I think you realize that we're lost and we should ask the way* contains a coordination of finite clauses serving as a complement within a clausal complement, but it could be re-expressed paratactically (*Here are my thoughts. You realize our situation. We're lost. We should ask the way*). Such re-expression transfers the (semantic) complexity of the content of the different clauses from the sentence level to the paragraph level. Absence of syntactic embedding and coordination resources in a language that calls for certain content to be expressed multisententially rather than unisententially is not the same as rendering a thought inexpressible or untranslatable.

Just as absence of syntactic support for infinitude claims about some language does not imply anything demeaning about its speakers, neither does it threaten the research program of transformational-generative grammar. Generative linguistics does not stand or fall with the infinitude claim. Overstatements like those in (1) or (2) can be dismissed without impugning any research progam. We argued at the end of § 3.2 above that generative rule systems with recursion do not *have* to be used to represent the syntax of languages with iterable subordination; but that does not mean it is an error to use generative rule systems for describing human languages or stating universal principles of linguistic structure. Whether it is an error or not depends on such things as the goals of a linguistic framework, and the data to which its theorizing aims to be responsive (Pullum and Scholz 2001, 2005).

## 6 Concluding remarks

Infinitude of human languages has not been independently established — and could not be. It does not represent a factual claim that can be used to support the idea that the properties of human language must be explicated via generative grammars involving recursion. Positing a generative grammar does not entail infinitude for the generated language anyway, even if there is recursion present in the rule system.

The remark of Lasnik (2000: 3), that "We need to find a way of representing structure that allows for infinity", therefore has it backwards. It is not that languages have been found to be infinite so our theories have to represent them as such. Language infinitude is not a reason for adopting a generative grammatical framework, but merely a theoretical consequence that will (under some conditions) emerge from adopting such a framework.

What remains true, by contrast, is Harris's claim (1957: 208) "If we were to insist on a finite language, we would have to include in our grammar several highly arbitrary and numerical conditions." No such arbitrary conditions should be added to grammars, of course. Ideally grammars should be stated in a way that insists neither on finitude nor on infinitude. It is a virtue of model-theoretic syntactic frameworks that they allow for this.

> School of Philosophy, Psychology and Language Sciences University of Edinburgh

## Notes

\* The authors are grateful for the support of fellowships at the Radcliffe Institute for Advanced Study in 2005–2006, during which some of the ideas set out here were conceived. A much earlier version of the paper was to have been presented at the conference on Recursion in Human Languages at Illinois State University in April 2007, but air travel problems prevented that, so the ideas presented here did not have the benefit of comments by the conference participants. We are very grateful to Julian Bradfield, Gerald Gazdar, Harry van der Hulst, András Kornai, Gereon Müller, Paul Postal, and four referees for comments on earlier drafts. These people should certainly not be assumed to agree with what we have said; the views presented here are ours alone, as are any and all errors. <sup>1</sup> Standard generative grammars cannot account for multiple coordination with unbounded branching degree. They enforce an undesired numerical upper bound on the number of coordinate daughters a node can have. The point is irrelevant to our theme here, so we henceforth ignore it; but see Rogers (1999) for a very interesting non-generative approach.

<sup>2</sup> The Axiom of Mathematical Induction, despite its suggestive name, is not relevant here. It states that if a set contains 1, and contains the successor of every member, then all the positive integers are members. The point is to rule out non-standard models of arithmetic, where there are additional offthe-scale integers, unreachable via successor. The two axioms mentioned in the text are sufficient to guarantee an infinity of integers.

<sup>3</sup> For various well-behaved theories of grammar the infinitude question is decidable, however. These include context-free-equivalent theories such as GPSG (Gazdar et al., 1985) and the formalization of 'minimalist' grammars developed by Stabler (1997).

<sup>4</sup> The notorious assertions that begin Montague (1970a) and Montague (1970b), to the effect that there are "no important theoretical difference between natural languages and the artificial languages of logicians", were shockingly at variance with the views of most linguists in 1970. And Montague does not appear to regard the mere availability of infinitely many expressions as significant fact about natural languages anyway: that is not what he was intending to emphasize.

### References

- Bach, Emmon. 1964. An Introduction to Transformational Grammars. New York: Holt Rinehart and Winston.
- Bach, Emmon. 1974. Syntactic Theory. New York: Holt Rinehart and Winston.
- Bever, Thomas G., Jerry A. Fodor, and Merrill Garrett. 1968. A formal limitation of associationism. In Theodore R. Dixon and David L. Horton, eds., *Verbal Behavior* and General Behavior Theory, 582–585. Englewood Cliffs: Prentice-Hall.
- Blackburn, Patrick and Claire Gardent. 1995. A specification language for lexical functional grammars. In *Proceedings of the 7th EACL*, 39–44. European Association for Computational Linguistics.
- Carnie, Andrew. 2002. Syntax: A General Introduction. Oxford: Blackwell.
- Chomsky, Noam. 1957a. Review of Charles F. Hockett, a manual of phonology. International Journal of American Linguistics 23:223–234.
- Chomsky, Noam. 1957b. Syntactic Structures. The Hague: Mouton.

- Chomsky, Noam. 1958. Linguistics, logic, psychology, and computers. In John Weber Carr, ed., Computer Programming and Artificial Intelligence, An Intensive Course for Practicing Scientists and Engineers: Lectures Given at the University of Michigan, Summer 1958, 429–456. Ann Arbor: University of Michigan College of Engineering.
- Chomsky, Noam. 1980. Rules and Representations. Oxford: Basil Blackwell.
- Chomsky, Noam. 1981. Lectures on Government and Binding. Dordrecht: Foris.
- Chomsky, Noam. 1986. *Knowledge of Language: Its Origins, Nature, and Use.* New York: Praeger.
- Chomsky, Noam. 2002. On Nature and Language. Cambridge: Cambridge University Press.
- Collinder, Björn. 1960. Comparative Grammar of the Uralic Languages. Stockholm: Almqvist & Wiksell.
- Derbyshire, Desmond C. 1979. *Hixkaryana*. Number 1 in Lingua Descriptive Series. Amsterdam: North Holland.
- Deutscher, Guy. 2000. Syntactic Change in Akkadian: The Evolution of Sentential Complementation. Oxford: Oxford University Press.
- Dixon, R. M. W. 1981. Wargamay. In R. M. W. Dixon and Barry J. Blake, eds., Handbook of Australian Languages, Volume 2, 1–144. Canberra: Australian National University Press.
- Dixon, Robert M. W. 1972. *The Dyirbal Language of North Queensland*. Cambridge: Cambridge University Press.
- Dretske, Fred I. 1965. Counting to infinity. Analysis 25:Supplement 3, 99-101.
- Epstein, Sam and Norbert Hornstein. 2005. Letter on 'The future of language'. Language 81:3-6.
- Evans, Gareth. 1981. Reply: Syntactic theory and tacit knowledge. In S. H. Holtzman and C. M. Leich, eds., *Wittgenstein: To Follow a Rule*, 118–137. London: Routledge and Kegan Paul.
- Everett, Daniel L. 2005. Cultural constraints on grammar and cognition in Pirahã: Another look at the design features of human language. *Current Anthropology* 46:621–646.
- Gazdar, Gerald, Ewan Klein, Geoffrey K. Pullum, and Ivan A. Sag. 1985. *Generalized Phrase Structure Grammar*. Oxford: Basil Blackwell.
- Ginzburg, Jonathan and Ivan A. Sag. 2000. *Interrogative Investigations*. Stanford: CSLI Publications.
- Givon, Talmy. 1979. On Understanding Grammar. New York: Academic Press.
- Harris, Zellig. 1957. Co-occurrence and transformation in linguistic structure. Language 33:283–340. Published version of a paper read to the Linguistic Society of America in 1955. The page reference is to the reprinting in Jerry A. Fodor and Jerrold J. Katz (eds.), *The Structure of Language: Readings in the Philosophy of Language*, 155–210 (Englewood Cliffs: Prentice-Hall).

- Harris, Zellig S. 1968. Mathematical Structures of Language. Interscience Tracts in Pure and Applied Mathematics, 21. New York: Interscience Publishers.
- Harwood, F. W. 1955. Axiomatic syntax: The construction and evaluation of a syntactic calculus. *Language* 31:409–413.
- Hauser, Marc D., Noam Chomsky, and Warren Tecumseh Fitch. 2002. The faculty of language: What is it, who has it, and how did it evolve? *Science* 298:1569–1579.
- Hockett, Charles F. 1955. A Manual of Phonology. Baltimore: Waverly Press.
- Huddleston, Rodney. 1976. An Introduction to English Transformational Syntax. London: Longman.
- Johnson, David E. and Paul M. Postal. 1980. Arc Pair Grammar. Princeton: Princeton University Press.
- Kaplan, Ronald. 1995. The formal architecture of lexical-functional grammar. In Mary Dalrymple, Ronald M. Kaplan, John T. Maxwell II, and Annie Zaenen, eds., *Formal Issues in Lexical-Functional Grammar*, 7–27. Stanford: CSLI Publications.
- Katz, Jerrold J. 1966. The Philosophy of Language. New York: Harper and Row.
- Katz, Jerrold J. 1978. Effability and translation. In F. Guenthner and M. Guenthner-Reutter, eds., *Meaning and Translation: Philosophical and Linguistic Approaches*, 191–234. London: Duckworth.
- Langacker, Ronald W. 1973. *Language and Its Structure*. New York: Harcourt Brace Jovanovich, second ed.
- Lashley, Karl. 1951. The problem of serial order in behavior. In L. A. Jeffress, ed., Cerebral Mechanisms in Behavior, 112–136. New York: Wiley.
- Lasnik, Howard. 2000. Syntactic Structures *Revisited: Contemporary Lectures on Classic Transformational Theory*. Cambridge, MA: MIT Press.
- Manaster Ramer, Alexis. 1993. Towards transductive linguistics. In Karen Jensen, George E. Heidorn, and S. D. Richardson, eds., *Natural Language Processing: The PLNLP Approach*, 13–27. Dordrecht: Kluwer Academic.
- Montague, Richard. 1970a. English as a formal language. In Bruno Visentini, ed., Linguaggi nella Società e nella Tecnica, 189–224. Milan: Edizioni di Comunità. Reprinted in Thomason (ed.) 1974.
- Montague, Richard. 1970b. Universal grammar. *Theoria* 36:373–398. Reprinted in Thomason (ed.) 1974.
- Pawley, Andrew and Frances Syder. 2000. The one clause at a time hypothesis. In Heidi Riggenbach, ed., *Perspectives on Fluency*, 163–191. Ann Arbor: University of Michigan Press.
- Pinker, Steven. 1994. The Language Instinct. New York: William Morrow.
- Pinker, Steven and Ray S. Jackendoff. 2005. What's special about the human language faculty? *Cognition* 95(2):201–236.
- Pollard, Carl. 1999. Strong generative capacity in HPSG. In Gert Webelhuth, Jean-Pierre Koenig, and Andreas Kathol, eds., *Lexical and Constructional Aspects of*

Linguistic Explanation, 281–297. Stanford: CSLI Publications.

Postal, Paul M. 1964. Underlying and superficial linguistic structure. *Harvard Educational Review* 34:246–266.

- Pullum, Geoffrey K. and Barbara C. Scholz. 2001. On the distinction between model-theoretic and generative-enumerative syntactic frameworks. In Philippe de Groote, Glyn Morrill, and Christian Retoré, eds., *Logical Aspects of Computational Linguistics: 4th International Conference*, number 2099 in Lecture Notes in Artificial Intelligence, 17–43. Berlin: Springer Verlag.
- Pullum, Geoffrey K. and Barbara C. Scholz. 2005. Contrasting applications of logic in natural language syntactic description. In Petr Hajek, Luis Valdes-Villanueva, and Dag Westerstahl, eds., *Proceedings of the 13th International Congress of Logic, Methodology and Philosophy of Science*. London: KCL Publications.
- Rogers, James. 1997. "Grammarless" phrase structure grammar. *Linguistics and Philosophy* 20:721–746.
- Rogers, James. 1999. The descriptive complexity of generalized local sets. In Hans-Peter Kolb and Uwe Mönnich, eds., *The Mathematics of Syntactic Structure: Trees and their Logics*, number 44 in Studies in Generative Grammar, 21–40. Berlin: Mouton de Gruyter.
- Rogers, James and Geoffrey K. Pullum. 2007. Aural pattern recognition experiments and the subregular hierarchy. To appear in UCLA Working Papers in Linguistics.
- Rumelhart, David and J. L. McClelland. 1986. On learning the past tenses of English verbs. In J. L. McClelland and D. E. Rumelhart, eds., *Parallel Distributed Processing: Explorations in the Microstructure of Cognition, Volume 2: Psychological and biological models*, 216–271. Cambridge, MA: MIT Press.
- Sampson, Geoffrey. 2009. A linguistic axiom challenged. In *Language Complexity* as an Evolving Variable. Oxford: Oxford University Press.
- Scholz, Barbara C. and Geoffrey K. Pullum. 2007. Tracking the origins of transformational generative grammar. *Journal of Linguistics* 43:701–723.
- Seuren, Pieter A. M. 2004. Chomsky's Minimalism. Oxford: Oxford University Press.
- Stabler, Edward. 1997. Derivational minimalism. In Christian Retoré, ed., Logical Aspects of Computational Linguistics, LACL '96, number 1328 in Lecture Notes in Artificial Intelligence, 68–95. Berlin: Springer Verlag.
- Stabler, Edward. 1999. Formal grammars. In Robert A. Wilson and Frank C. Keil, eds., *The MIT Encyclopedia of the Cognitive Sciences*, 320–322. Cambridge, MA: MIT Press.
- Yang, Charles. 2006. The Infinite Gift. New York: Scribner.